(Non-)Geometry of SCFTs

山崎 雅人
(Princeton)

Apr/17/2013 @ Texas A&M

"Topics in Holography, Supersymmetry and Higher Derivatives"

Supersymmetry IR fixed pt

(Non-)Geometry of SCFTs

Supersymmetry

IR fixed pt

(Non-)Geometry of SCFTs

a class of SCFTs

geometric / algebraic / combinatorial structures

Supersymmetry

IR fixed pt

(Non-)Geometry of SCFTs

a class of SCFTs

"a new paradigm to understand SCFTs"
(and old)

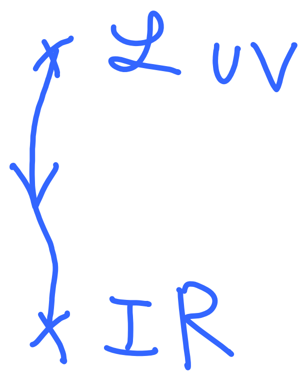
IR fixed pts are subtle

a) strongly coupled
non-pert. effects

$$\begin{tikzcd} \times \ \mathcal{L} \ \mathrm{UV} \arrow[d] \\ \times \ \mathrm{IR} \end{tikzcd}$$

IR fixed pts are subtle

a) strongly coupled
non-pert. effects



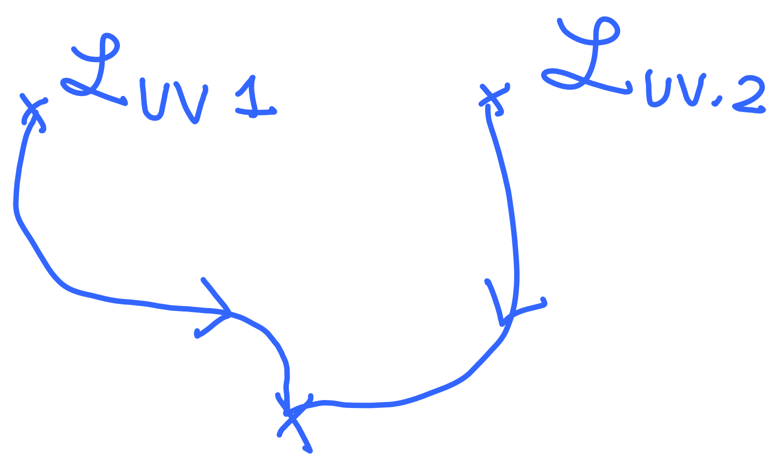
b) IR dualities

e.g. 4d $\mathcal{N}=4$

S-dual $\tau \to \frac{a\tau+b}{c\tau+d}$

4d $\mathcal{N}=1$ Seiberg dual

3d $\mathcal{N}=2$ mirror etc.



IR fixed pts are subtle

a) strongly coupled
non-pert. effects

$$\mathcal{L}_{UV} \longrightarrow IR$$

c) non-Lagrangian

UV $\cancel{\mathcal{L}}$

RG by relevant deformation

IR

b) IR dualities

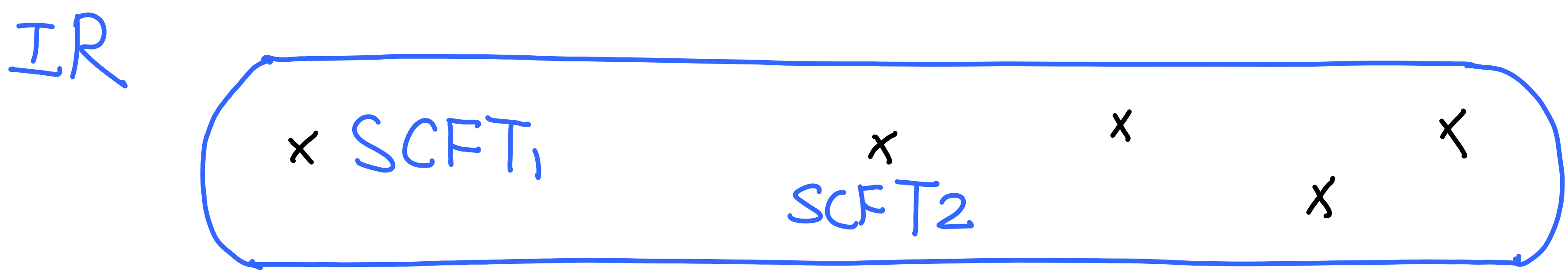
e.g. 4d $\mathcal{N}=4$

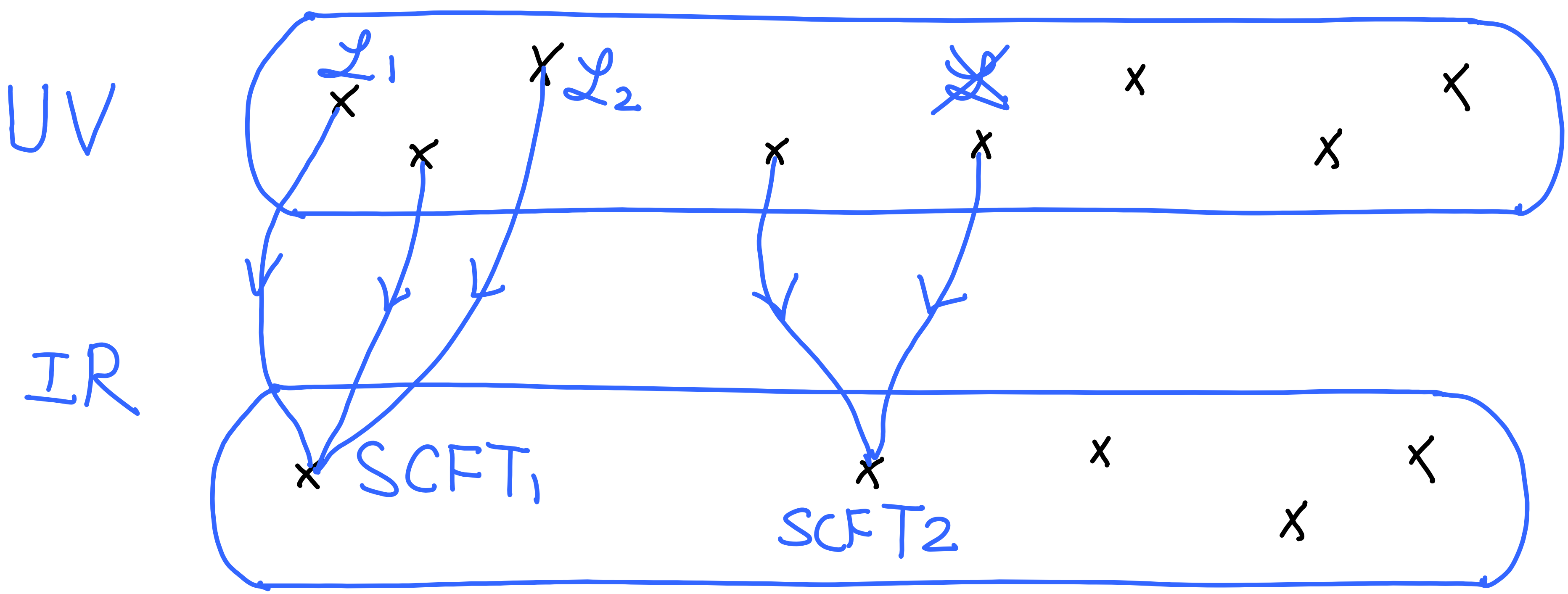
S-dual $\tau \to \dfrac{a\tau+b}{c\tau+d}$

4d $\mathcal{N}=1$ Seiberg dual

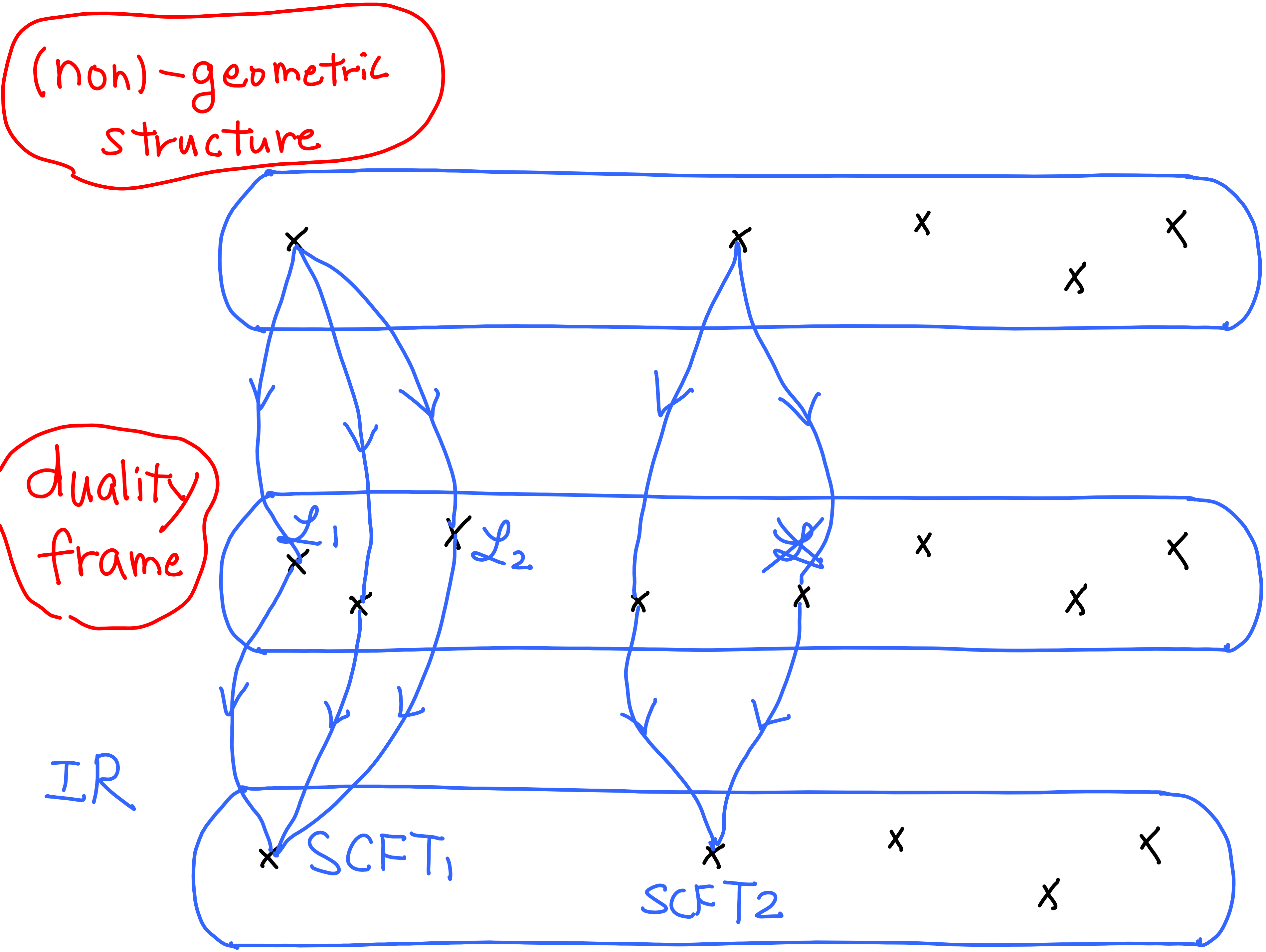
3d $\mathcal{N}=2$ mirror etc.

$$\mathcal{L}_{UV\,1} \longrightarrow \times \longleftarrow \mathcal{L}_{UV,2}$$


IR
SCFT$_1$
SCFT$_2$



UV
IR
$\mathcal{L}_1$
$\mathcal{L}_2$
SCFT$_1$
SCFT2



(non)-geometric structure
duality frame
IR
$\mathcal{L}_1$
$\mathcal{L}_2$
SCFT$_1$
SCFT2

√ * Introduction

→ * Examples

- (non-)geometric structures in SCFT
- AGT-type relations

* An example in detail

SCFT	duality frame	duality	(non)geometric structure
1. 4d $\mathcal{N}=2$	pants decomposition	4d $\mathcal{N}=2$ S-duality	2-mfd Σ
2. 3d $\mathcal{N}=2$	ideal triangulation	3d mirror sym.	3-mfd M
3. 4d $\mathcal{N}=1$ 3d $\mathcal{N}=2$	quiver	4d Seiberg dual 3d Giveon-Kutasov dual	CY3 X_3 CY4 X_4
4. 4d $\mathcal{N}=1$	bicolored graph	4d $\mathcal{N}=1$ Seiberg dual	cell of $(\mathrm{Gr}_{k,n})_{\geq 0}$
5. 3d $\mathcal{N}=2$	mutation sequence	3d mirror sym.	change of quivers & clusters

later

1. [Witten('97) Gaiotto('09)]

SCFT

4d $\mathcal{N}=2$

$T[C]$

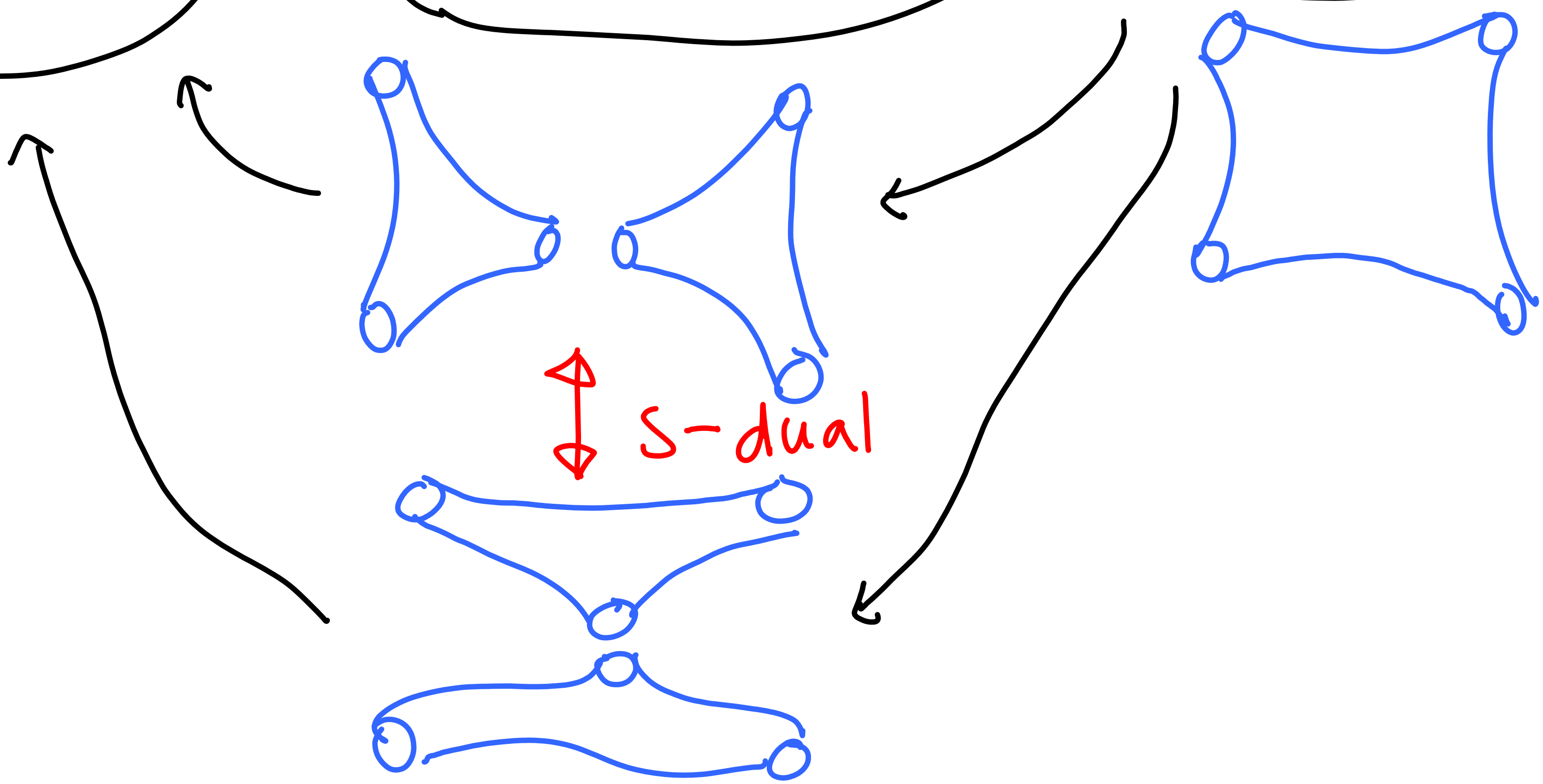
duality frame

pants decomposition

geometry

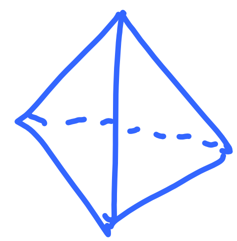
2-mfd

C



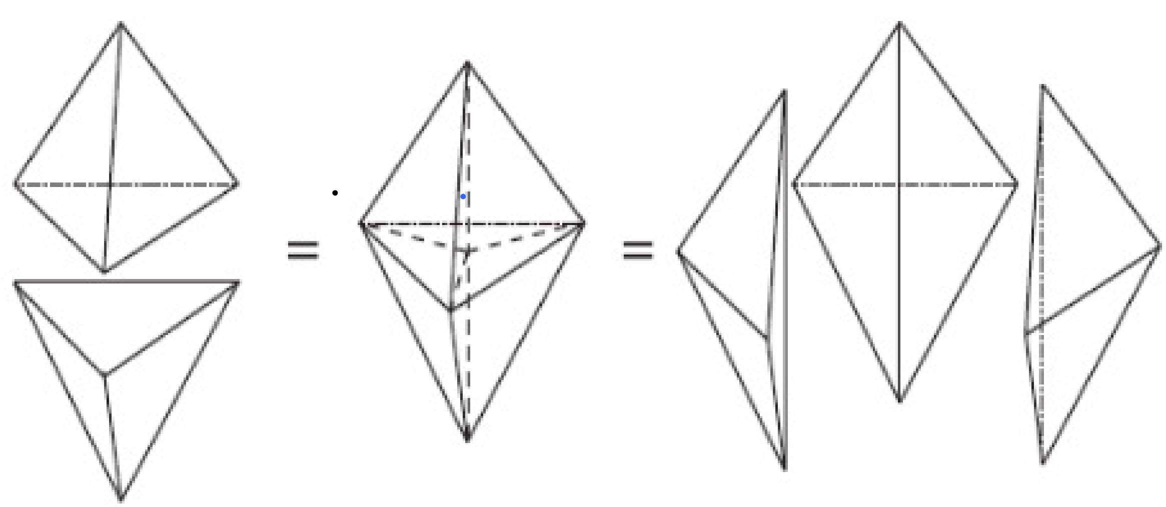
2. [('11~) Terashima-Y; Dimofte-Gaiotto-Gukov; Cecotti-Cordova-Vafa, $\cdots$]

SCFT 3d $\mathcal{N}=2$ $T[M]$ $\leftarrow$ duality frame 3d (ideal) triangulation $\leftarrow$ geometry 3-mfd M

$M = \bigcup$ 

2. [('11–) Terashima–Y; Dimofte–Gaiotto–Gukov; Cecotti–Cordova–Vafa, ⋯]

SCFT		duality frame		geometry
3d $\mathcal{N}=2$ $T[M]$	$\leftarrow$	3d (ideal) triangulation	$\leftarrow$	3-mfd M



2–3 Pachner move
||
3d mirror sym

3. AdS/CFT examples [Douglas-Moore, ... many papers]

$$\begin{tikzcd}
\text{SCFT: 4d } \mathcal{N}=1 \text{ theories} & \text{duality frame: quiver} \arrow[l] & \text{(toric) } CY_3\text{: } X_3 \arrow[l] \\
\text{3d } \mathcal{N}=2 \text{ CSm theories} & \text{quiver + CS level} \arrow[l] & \text{(toric) } CY_4\text{: } X_4 \arrow[l]
\end{tikzcd}$$

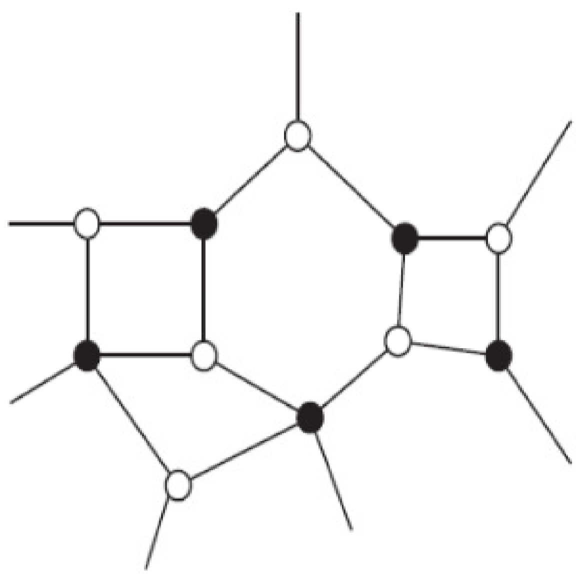
4d $\mathcal{N}=1$ Seiberg dual

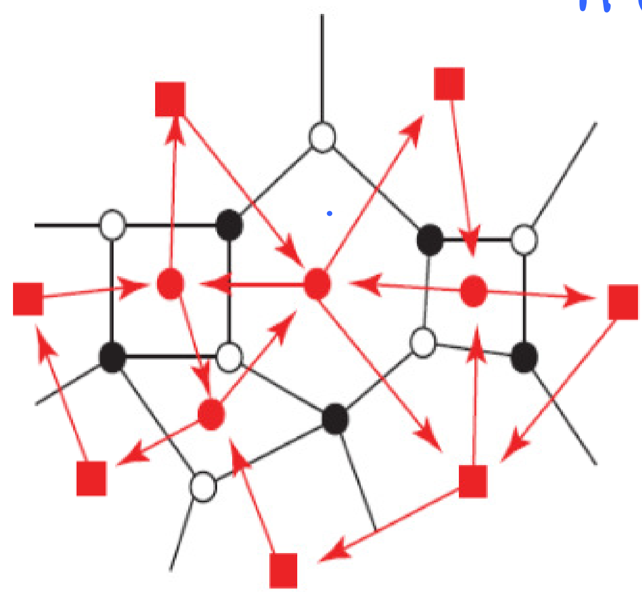
3d $\mathcal{N}=2$ Giveon-Kutasov dual

[※ "decomposition" unclear ...]

4. [('12) Xie–Y; Franco; stringy realization by Heckman–Vafa–Xie–Y]

$$\begin{array}{ccccc} \text{SCFT} & & \text{duality frame} & & \\ \text{4d } \mathcal{N}=1 & \longleftarrow & \text{planar bicolored graph} & \longleftarrow & \text{a cell of } (Gr_{k,n})_{\geq 0} \\ & & \uparrow & & \text{or permutation } \pi \in \mathfrak{S}_n \\ & & \text{4d } \mathcal{N}=1 \text{ Seiberg dual} & & \end{array}$$





SCFT	duality frame	duality	(non)geometric structure
1. 4d $\mathcal{N}=2$	pants decomposition	4d $\mathcal{N}=2$ S-duality	2-mfd Σ
2. 3d $\mathcal{N}=2$	ideal triangulation	3d mirror sym.	3-mfd M
3. 4d $\mathcal{N}=1$ 3d $\mathcal{N}=2$	quiver	4d Seiberg dual 3d Giveon-Kutasov dual	CY3 X_3 CY4 X_4
4. 4d $\mathcal{N}=1$	bicolored graph	4d $\mathcal{N}=1$ Seiberg dual	cell of $(Gr_{k,n})_{\geq 0}$
5. 3d $\mathcal{N}=2$	mutation sequence	3d mirror sym.	change of quivers & clusters

top-down vs. bottom-up string theory?

SCFT
$T[X]$

(non-) geometric structure
"X"

"$Z[T[X]] = Z[X]$"

computed by localization

1.

$$Z^{4d\ \mathcal{N}=2}_{T[\Sigma]}[S^4_b] = \left\langle \text{Liouville/Toda correlator} \right\rangle_\Sigma$$

[Alday Gaiotto Tachikawa ('09, ...)
Wyllard]

$$I^{4d\ \mathcal{N}=2}_{T[\Sigma]}[S^1 \times S^3] = \left\langle \text{2d TQFT (2d q-YM)} \right\rangle_\Sigma$$

superconformal index

[Gadde Pomoni Rastelli Razamat ('09)

Gadde Rastelli Razamat Yan ('11)]

2. $Z^{3d\ \mathcal{N}=2}_{T[M]}[S^3_b] = Z_{SL(N)\ CS}[M]$

[Terashima-Y; Dimofte-Gaiotto-Gukov, ...]

3. $I^{4d\ \mathcal{N}=1}_{\text{gauge}}[S^1 \times S^3] \xrightarrow[\text{large } N]{} I_{\text{gravity}}[SE_5]$

$I^{3d\ \mathcal{N}=2}_{\text{gauge}}[S^1 \times S^2] \xrightarrow[\text{large } N]{} I_{\text{gravity}}[SE_7]$

[many papers in AdS/CFT;
cf. recent paper by
Eager-Tachikawa-Schmude]

3 & 4

$$I^{4d\ \mathcal{N}=1}[S^1 \times S^3] = \mathcal{Z}^{2d}_{\text{integrable spin chain}}$$

[Y; Terashima-Y ('12)]

5.

$$\mathcal{Z}^{3d\ \mathcal{N}=2}_{T[Q,m]}[S^3] = \mathcal{Z}_{\text{cluster}}[Q,m]$$

✓ * Introduction

✓ * Examples

- (non-)geometric structures in SCFT
- AGT-type relations

→ * An example in detail

One of the most radical examples:

3d $\mathcal{N}=2$ theories $T[(Q, m)]$

defined from

a quiver & mutations

Q $\qquad$ m

[Terashima-Y ('13)]

based on cluster algebras

[Fomin Zelevinski, ...]

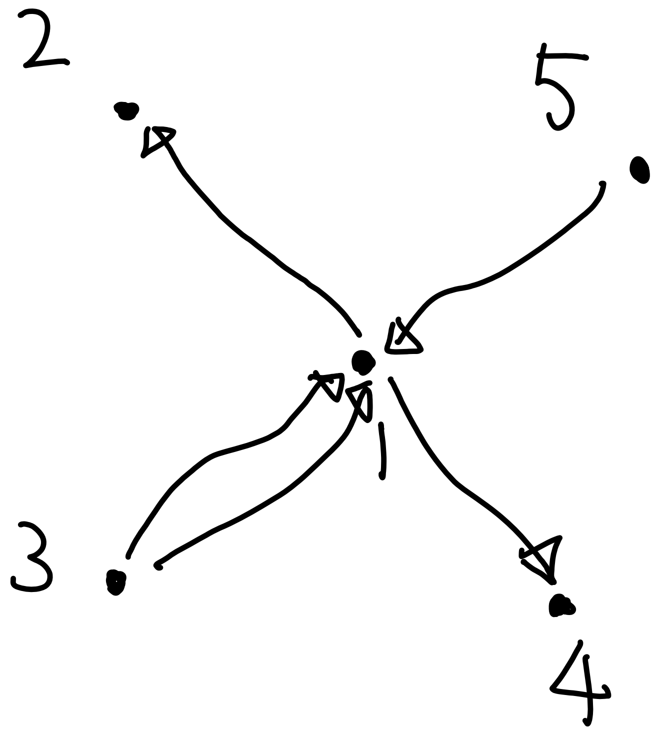
and [Kashaev-Nakanishi, Nagao-Terashima-Y ('11)]

Quiver : oriented graph Q

described by an antisymmetric matrix

$$Q_{i,j} = \#\{i \to j\} - \#\{j \to i\}$$

$$i,j \in I = \{\text{vertices of } Q\}$$



e.g. $Q_{12} = +1$

$Q_{15} = -1$

$Q_{13} = -2$

Q : quiver

$\rightsquigarrow$ $\mathcal{A}_Q$: space generated by y_i $(i \in I)$

with relation

$$y_j y_i = q^{2Q_{ij}} y_i y_j$$

$$q = e^{i\hbar} = e^{i\pi b^2}$$

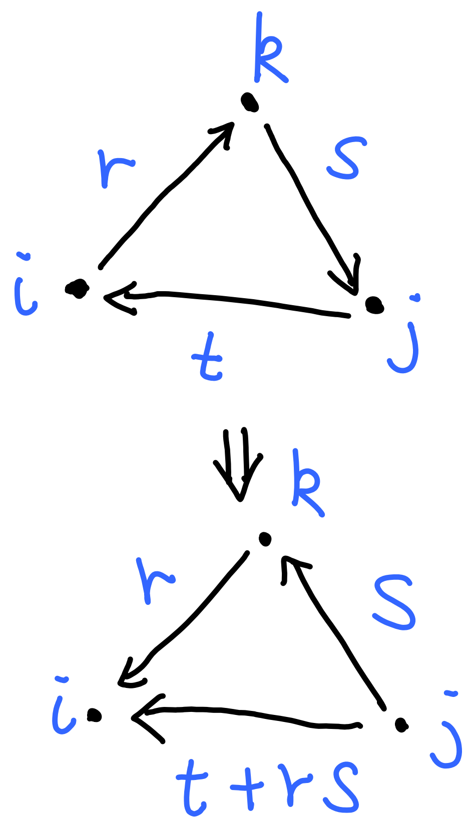
(or $[Y_i, Y_j] = 2Q_{ij}$ for $y_i = e^{Y_i}$ $\rightsquigarrow$ $[x_i, p_j] = i\hbar\delta_{ij}$, $[x_i, x_j] = [p_i, p_j] = 0$)

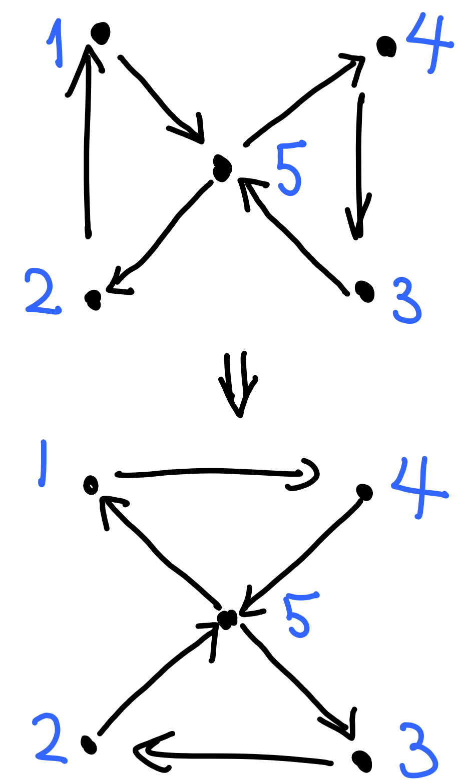
has standard repr. on Hilbert space $\mathcal{H}_Q$

$\mu_k Q$: mutation of quiver Q at vertex k

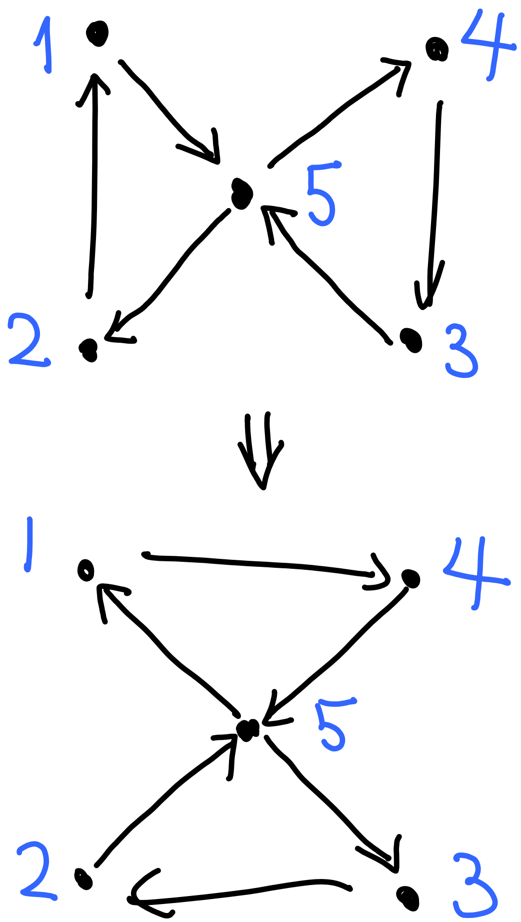
$$(\mu_k Q)_{ij} := \begin{cases} -Q_{ij} & (i=k \text{ or } j=k) \\ Q_{ij} + [Q_{ik}]_+ [Q_{kj}]_+ - [Q_{jk}]_+ [Q_{ki}]_+ & (i,j \neq k) \end{cases}$$

$([x]_+ := \max(x,0))$





mutation at vertex k of quiver Q $\rightsquigarrow$ $\hat{\mu}_k : \mathcal{A}_Q \to \mathcal{A}_{\mu_k Q}$



$$\begin{cases} y_1' = y_1 (1 + q y_5) \\ y_2' = y_2 (1 + q y_5^{-1})^{-1} \\ y_3' = y_3 (1 + q y_5) \\ y_4' = y_4 (1 + q y_5^{-1})^{-1} \\ y_5' = y_5^{-1} \end{cases}$$

quiver Q + a chain of mutations

$$m = (m_1, m_2, \cdots, m_L)$$

$\rightsquigarrow$ "cluster partition function"

$$Z_{(Q,m)} := \langle \mathrm{in} | \hat{\mu}_{m_1} \hat{\mu}_{m_2} \cdots \hat{\mu}_{m_L} | \mathrm{out} \rangle$$

$$|\mathrm{in}\rangle \in \mathcal{H}_Q$$

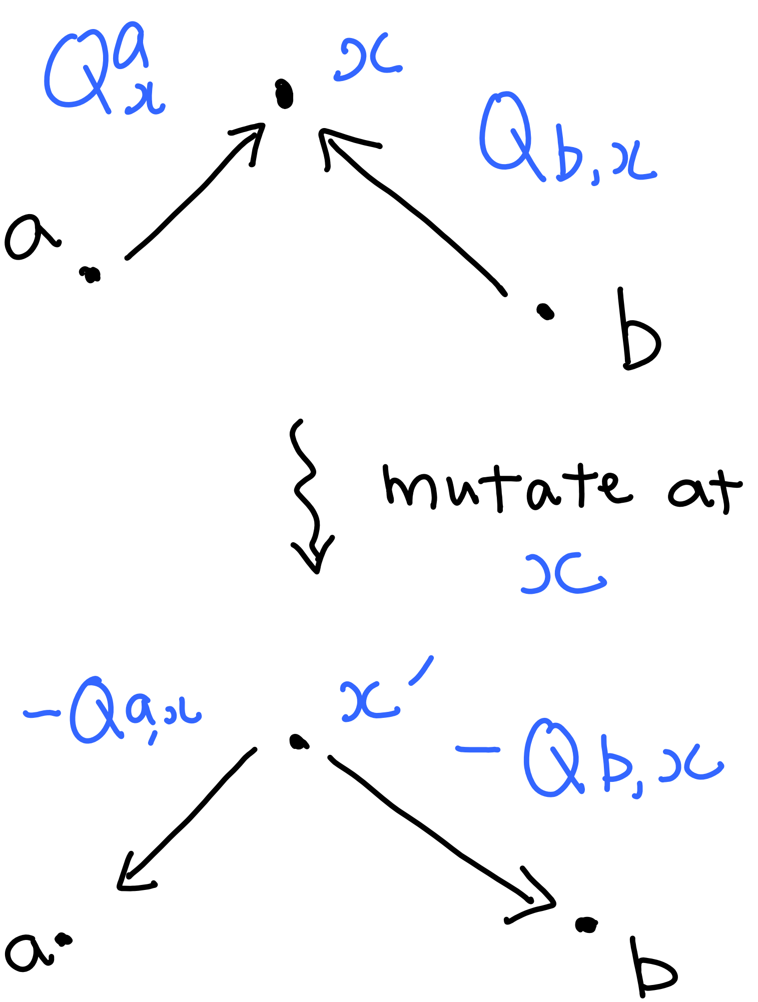
$$|\mathrm{out}\rangle \in \mathcal{H}_{\mu_L \cdots \mu_1 Q}$$

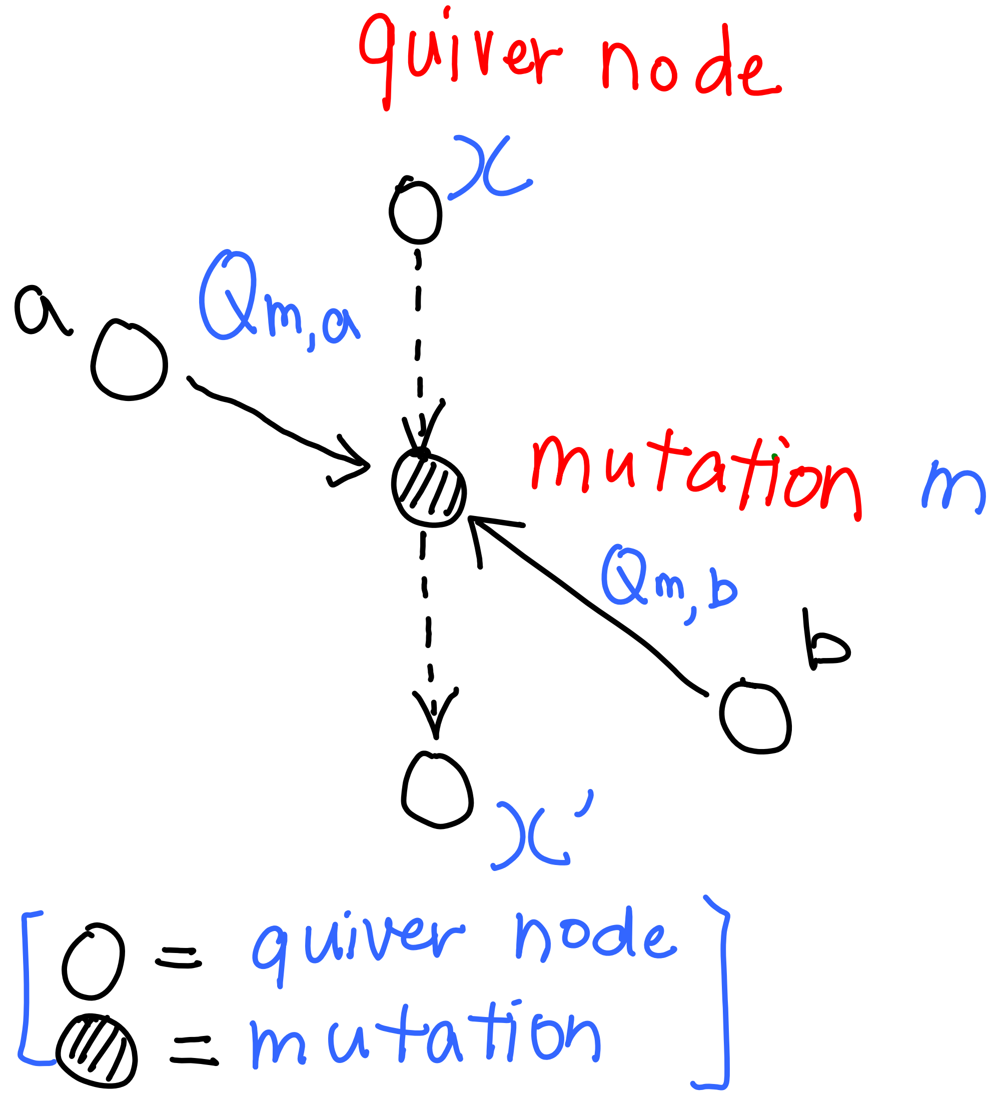
Claim:

We can construct 3d $\mathcal{N}=2$ theories $T[(Q,m)]$

s.t. $Z_{T[(Q,m)]}[S^3_b] = Z_{(Q,m)}$

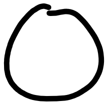
Useful to represent (Q, m) by mutation networks





1.  = mutation of quiver

= $\mathcal{N}=2$ hyper mult.

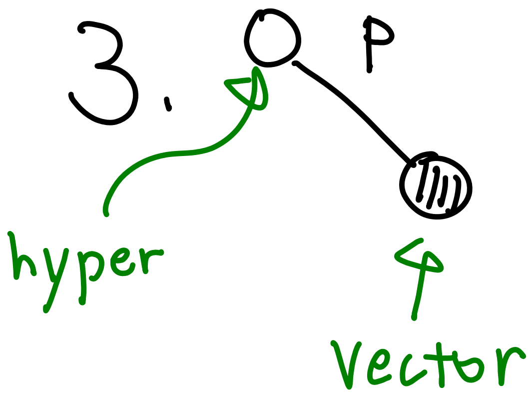
2.  = quiver vertex

= $U(1)$ symmetry $\begin{cases} \text{gauge} \\ \text{global} \end{cases}$

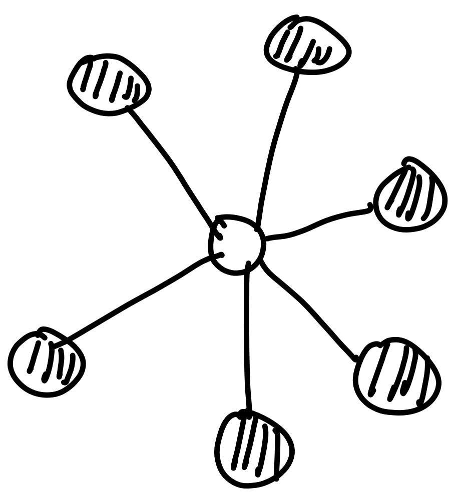
[$\mathcal{N}=2$ vector mult.]

y-variable = Wilson-'t Hooft loop operator

(※ many redundancies : electric/magnetic dual)

3. 

$=$ $\mathcal{N}=2$ hyper mult. has charge p under the $U(1)$ sym.

4. 

$=$ Superpotential constraint

[※ in general involves monopole operators]

Lesson:

$U(1)$ symmetries

& the algebra of

Wilson-'t Hooft operators

(almost) determines

3d $\mathcal{N}=2$ Abelian theories

Summary

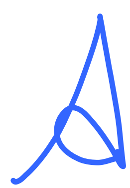
- IR fixed pts of SUSY field theories

  (and their IR dualities)

  from (non-)geometric structures

  $$\text{"}X\text{"} \rightsquigarrow \text{theory } T[X]$$

- 3d $\mathcal{N}=2$ example:

  $X \sim$  Wilson-'t Hooft