Towards a holographic formulation of cosmology

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Topics in holography, supersymmetry and higher derivatives  
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During the last century, Cosmology has seen some of the most spectacular discoveries in the history of Physics:

- **Big Bang**
- **Inflation**
- **Dark Energy**
- **Accelerated Expansion**

**Accelerating universe, both in the far past and future**

Simplest example: **de Sitter universe**, \( a \propto \exp(Ht) \)

de Sitter (1917)
Fascinating properties of accelerating cosmologies:

- Event horizons
- Thermodynamic laws

\[ T = H^{-1}, \quad S = \frac{\text{Area}}{4G_N} \]

Hawking, Bekenstein, Gibbons,... (70's)

- Lead to deep questions in theoretical cosmology:
  
  a) describe causal patch or global spacetime?
  b) stat. mech. origin of horizon thermodynamics?
  c) microscopic degrees of freedom? \( S_{dS} \sim 10^{120} \) ??

Formulate quantum gravity on cosmological spacetimes?!
Black holes also have event horizons and obey thermodynamic laws. The study of BHs led to a revolution in understanding quantum gravity in string theory: D-branes and AdS/CFT.

Revolution in understanding quantum gravity in string theory: D-branes and AdS/CFT

Polchinski (95); Maldacena (97)

Is holography a general property of quantum gravity? Holographic duality for cosmology?

‘t Hooft (93); Susskind (94)
Why expect a holographic description of dS?

✓ dS entropy = Area of cosmological horizon
✓ Basic mechanism responsible for AdS/CFT also present in de Sitter

\[ E_{\text{loc}} = \sqrt{-g_{00}} E_{pr} \ll M_{Pl} \]

gravitational redshift

Maldacena
To understand this, slice AdS and dS by dS

\[
\text{AdS}_{d+1} / dS_d
\]

from analytic continuation

\[
S^{d+1} / S^d, \quad ds^2 = d\theta^2 + \sin^2 \theta d\Omega^2
\]

\[
ds_{d+1}^2 = dr^2 + \sinh^2 \left( \frac{r}{R} \right) ds_d^2
\]

\[
E_{loc} \sim \frac{r}{R} E_{pr} \to 0
\]

Holographic dual: QFT living on dS_d

\[
ds_{d+1}^2 = dr^2 + \sin^2 \left( \frac{r}{R} \right) ds_d^2
\]

\[
E_{loc} \sim \frac{r}{R} E_{pr} \to 0
\]

Holographic dual: 2 EFTs on dS_d

Alishahiha, Karch, Silverstein, Tong
Our goal:

formulate cosmology holographically and determine the degrees of freedom that build up cosmological spacetimes

- **Strategy:** generalize AdS/CFT to cosmology
A. Uplifting AdS/CFT to cosmology

\[ \left( \frac{dR}{dr} \right)^2 - \frac{1}{R^2} - g_s^2 \left( \frac{N}{R^n} \right)^2 = 0 \]

Color branes

\[ \mathcal{R} > 0 \]

AdS/CFT duality

Dong, Horn, Silverstein, GT

AdS

\[ 1/R \]

Color flux
✓ Internal space of negative curvature obtained by adding magnetic flavor branes (more on this below)
✓ Orientifolds provide negative tension for the intermediate term in the potential
✓ With other ingredients in place to stabilize all light fields

... can lead to metastable de Sitter solutions

Concrete brane construction given in Dong, Horn, Silverstein, GT arXiv:1005.5403
Brane construction

Background internal geometry:

$$\mathcal{R} < 0$$

$$\left(\frac{dR/\,dr}{R^2}\right)^2 = -\frac{1}{R^2} + \frac{\text{const}}{R^{d\perp}}$$

- The cone of AdS/CFT now becomes compact
- Two tips; place color branes at one and antibranes at the other

Two large N QFTs on dS\(_d\) plus Gravity\(_d\)

\[dS_{d+1} \times \mathcal{B} \equiv \text{dS}/\text{dS}\]
Properties of the duality

- Two large N EFTs living on dS, at finite temperature $T = 1/R_{dS}$
- EFTs coupled to each other at $E \sim 1/R_{dS}$.
- Semiholographic: lower dimensional dynamical gravity. But screened by large N matter sector

Microscopic explanation of dS entropy

\[
\text{Area of dS horizon} \overset{12}{=} \text{Thermal entropy of large N matter sector}
\]

✓ parametric count
✓ uplifting ingredients subleading

FRW decays

Dong, Horn, Matsuura Silverstein, GT
arXiv:1108.5732

- Dual to two t-dep QFTs
- Precise duality at late times!

$S \to \infty$, $GR_d$ decouples
B. Magnetic flavors and cosmology

Consider \( AdS_5 \times S^5 \) from N D3 branes

\((p,q)\) 7 branes compete with internal curvature

- Wrap \( AdS_5 \times \Sigma_3 \)
- Large tension \( T \sim \frac{1}{R^2} \cdot \frac{1}{g_s^2} \)

\[ V_R \sim \frac{\Delta n}{R^2} , \quad \Delta n \equiv n_7 - n_* \]

\[ \Delta n < 0 : \text{AdS} \]

\[ \Delta n \geq 0 : \text{Cosmology} \]

- No static sols, but ...
- controlled t-dep FRW sols

Dual gauge theory w/ dyonic matter

Seiberg, Witten; Banks, Douglas, Seiberg; Aharony et al; Polchinski, Silverstein

arXiv:1108.5732
Dong, Horn, Matsuura Silverstein, GT

FRW holography
\[ \Delta n < 0 \text{ vs } \Delta n > 0 \text{ in dual QFT?} \]

\[ \Delta n < 0 \] can be traced to a unitarity bound

E.g. in Seiberg-Witten theory,

\[ \text{Dim}(u) \geq 1 \Rightarrow N_{\text{flavors}} < 12 \]

So in the static theory \( \Delta n > 0 \) would violate unitarity. Cannot happen in UV complete QFT!

Instead, hitting the unitarity bound \( \Rightarrow \)

- sector of theory becoming free
- possible transition away from CFT phase

However, for \( \Delta n > 0 \) there is a consistent gravity solution

- well-defined t-dep holographic dual

\[ \Rightarrow \text{ What happens to RG and unitarity bounds in QFTs with spacetime dependent couplings?} \]
Intuition:  
\[ S = S_{CFT} + \int d^d x \lambda_0 |x|^\alpha \mathcal{O} \]  
\[ \text{Dim}(\mathcal{O}) = \Delta \text{ at } \lambda_0 = 0 \]  

- Power counting:  \[ \text{Dim}\lambda_0 = d - \Delta + \alpha \Rightarrow \text{relevant for } \alpha \geq \Delta - d \]  

Tractable class of theories: “semiholographic” model  
\[ S = S_{CFT} - \int d^d x \frac{1}{2} ((\partial \phi)^2 + m^2 \phi^2) + \int d^d x g(x) \mathcal{O} \phi \]  
Large N sector  
\[ \text{at } g = 0, \text{ Dim}(\mathcal{O}) = \frac{d}{2} + \nu \]  
i) Static:  \[ \rightsquigarrow \text{Dim}(g) = 1 - \nu \text{ irrelevant for } \nu > 1 \]  
ii) Spacetime dep:  \[ g(x) = g_0 |x|^\alpha \text{ relevant for } \alpha > \nu - 1 \]  

arXiv:1203.1680 Dong, Horn, Silverstein, GT
At large N, effectively gaussian ... even with \( g(x) \! \)

\[
g(x) = g_0 |x|^\alpha
\]

\( \alpha > \nu > 1 \)

\[
E \sim \frac{g'}{g} \sim \frac{1}{x}
\]

**Energy**

**Landau pole**

**UV completions**

\( \checkmark \phi(x) \) composite of gauge theory (e.g. SQCD)

**nonadiabatic effects**

\[
\langle \phi(x)\phi(x') \rangle \sim \frac{1}{g(x)|x - x'|^{d-2\nu} g(x')}
\]

\[
\text{Dim}(\phi) = \frac{d}{2} - \nu + \alpha
\]

**Infrared**

... corrections at very late times

Translation breaking but new approx. scale invariant fixed point!
C. RG structure of dS dual

Conditions on a QFT so that it can describe holographically an observer patch of de Sitter?

From gravity side, both AdS and dS look very special ...

✓ maximal symmetry  \[ ds_{d+1}^2 = dr^2 + a(r)^2 ds_{dS}^2 \]

✓ moduli stabilization  \[ V'(\phi) = 0 \]

For AdS, properties should be equivalent to having a CFT

Features of dS dual that encode max. sym. and stabilization?

Dong, Horn, Silverstein, GT
arXiv:1209.5392
Holographic RG

Energy-Radius relation:

\[ ds^2 = dr^2 + a(r)^2 \hat{g}_{\mu \nu} dx^\mu dx^\nu \Rightarrow E_{\text{loc}} = a(r) E_{\text{proper}} \]

\[
Z_{\text{QFT}} = \int \mathcal{D}M e^{-S_{\text{QFT}}} = \\
\int \mathcal{D}M|_{E<E_L} \left( \int \mathcal{D}M_{E>E_L} e^{-S_{\text{QFT}}} \right) \\
e^{-S_{\text{eff}}(E_L)}
\]

\[
Z_{\text{bulk}} = \int \mathcal{D}\phi e^{-S} = \\
\int \mathcal{D}\tilde{\phi} \left( \int \mathcal{D}\phi|_{r<L} e^{-S|_{r<L}} \right) \left( \int \mathcal{D}\phi|_{r>L} e^{-S|_{r>L}} \right) \\
\Psi_{IR}(L, \tilde{\phi}) \quad \Psi_{UV}(L, \tilde{\phi})
\]

Postulate: \[
\Psi_{IR}(L, \tilde{\phi}) = \int \mathcal{D}M|_{E<E_L} e^{-S_0(M)+\int d^d x \sqrt{g} \tilde{\phi} \mathcal{O}}
\]

\[
e^{-S_{\text{eff}}(E_L)} = e^{-S_0} \int \mathcal{D}\tilde{\phi} e^{\int d^d x \sqrt{g} \tilde{\phi} \mathcal{O}} \Psi_{UV}(L, \tilde{\phi})
\]
Consequences of moduli stabilization

Bulk scalar sitting at (local) minimum: \( V'(\phi_*) = 0 \)

\[
S = - \int d^{d+1}x \sqrt{g} \left( (\partial \phi)^2 + V(\phi) + \ldots \right) , \quad V(\phi) = V_* + \frac{1}{2} V''(\phi - \phi_*)^2 + \ldots
\]

\( \Psi_{UV} \) dominated by classical trajectory \( \phi(L_{UV}) = \phi_* \rightarrow \phi(L) = \bar{\phi} \)

\[ \Rightarrow \log \Psi_{UV} \text{ has no linear term in } \bar{\phi} - \phi_* \]

Wilson action \( S_{\text{eff}} = S_0(M) + g_1 \mathcal{O} + \frac{1}{2} g_2(L) \mathcal{O}^2 + \ldots \)

1. Single trace couplings do not run, \( \partial_L g_1 = 0 \)

2. Iterative structure of RG: \( \partial_L g_n \) only depends on \( g_{m<n} \)

Strong simplification of RG evolution

Suggests new way of organizing the QFT path integral!
D. Summary and future directions

- dS and FRW dual to two EFTs plus gravity; brane construction and dS/dS. Parametric explanation of entropy.

- matter content: magnetic flavors, uplifting, unitarity. t-dep, RG and shifts in unitarity bounds

- special properties of dS dual; moduli stabilization and max. sym. Strong simplification of RG.

Nontrivial agreement between micro and macroscopic results indicates a consistent framework for holographic cosmology.
Future directions

- Develop more string theory constructions of dS and FRW

- Explicit description of matter content and couplings of dual QFTs
  ➡ class of QFTs dual to de Sitter

- Is uplifting the only mechanism for obtaining a dS dual?
  Relation to global dS/CFT description?

- Reorganization of path integral according to holographic RG
  ➡ meaning of radial cutoff
  ➡ structure of beta functions

- New critical phenomena from spacetime dependent couplings
  ➡ physical generalization of Wilson-Fisher in integer dimensions
  ➡ applications to controlled models of high Tc superconductors

Thank you!