M-theory and Type IIA Flux Vacua

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Outline:

I Introduction

II New Sources of B-type Charge

III Examples
Introduction

- Basic no-go theorem forbidding flux compactifications in M-theory and string theory

Consider 11-dim G2 BRA:

\[ S_{11} = \frac{1}{2k^2} \int d^{11}x \sqrt{-G} \left( R - \frac{1}{4} G^{11} \right) - \frac{1}{2k^2} \int d^{11}x \sqrt{-G} \nabla \nabla G \]

Take a warped product \( M_7 \times M_4 \times \mathbb{R} \):

\[ ds^2 = e^{2w(y)} \left( \tilde{g}_{\mu\nu} dx^\mu dx^\nu + \tilde{g}_{mn}(y) dy^m dy^n \right) \]

Lorentz \( = \tilde{g}_{\mu\nu} \), \( g_{mn} = f \sqrt{-\tilde{g}(y)} \)

No sources \( \Rightarrow \) \( 8 \beta = 0 \) and \( f = f_0 e^{-4w} \)

E.o.m.: \( R_{mn} = \frac{1}{2} \left( \tilde{g}_{mn} G - 2 \tilde{g}_{mn} \nabla^2 \right) \)

\[ d^* \nabla G = -\frac{1}{2} \nabla G \]
Maximally symmetric space-time:

Adding or Mink. requires \( \hat{F}_{\mu \nu} = \frac{1}{2} \varphi \delta_{\mu \nu} \)

\( w/ w \) const.

\( R^{(4)} = -\frac{\mu}{3} \varphi^2 - \frac{2}{3} \left[ 6 \hat{\psi} \psi \right] \)

from sources

\( R^{(2)} = \frac{5}{6} \left[ 6 \hat{\psi} \psi \right] + \frac{2}{6} \left( \frac{\delta_{\mu \nu}}{\text{dim.}} \right) \)

\( = \frac{5}{6} \left[ 6 \hat{\psi} \psi \right] \)

\( \Rightarrow R^{(4)} \leq 0 \)

\( = \frac{5}{6} \left( R^{(4)} - 4 \hat{\psi} \psi - 36 \left[ \delta_{\mu \nu} \right] \right) \)

\( = \frac{4}{5} \frac{c^{-2w}}{m^2} \left( \lambda - \frac{1}{4} \frac{c^{-2w}}{m^2} \lambda^2 \right) \)

\( \lambda = \frac{\int m^2 c^{-2w} R^{(4)} \, dv}{4 \int m^2 c^{-2w} \, dv} \)

So \( \lambda = 0 \Rightarrow R^{(4)} = 0 \Rightarrow \text{All } 6+1 \text{vectors vanish.} \)

No Mink. or de Sitter.

\( \hat{F}_{\mu \nu} = \hat{\psi} \psi \hat{F}_{\mu \nu} = \hat{\psi} \psi \left( \hat{\psi} \psi + 9 \hat{\psi} \psi \right) \)

\( = \frac{7}{144} \int e^{-6w} - \frac{1}{144} e^{2w} \left[ 6 \hat{\psi} \psi \right] \left[ \left[ \lambda \right]^2 \right] \)

Strongest constraint since the r.h.s. must be point-wise independent.
Framed - Rubin $A_5$ sols correspond to $w \equiv 0$. There are a few generically above.

Generically $\Lambda$ is a (curvature $R^{(2)}$). Flux solns $\Lambda \ll R^{(2)}$ parametrically smaller than $R^{(2)}$ encounter the Mink. no-go thm. None known (to my knowledge).

Evaing this no-go (Gibbons) requires higher derivative ingredients.
No mechanism is known in m-theory, IIA or IIB. I will comment on massive IIA later.

In HeV/Type D, there are $R^2$ couplings

$$\int \sqrt{g} \, R + R^2 + \ldots$$

$$d \mathcal{L} = d^4 \theta \left( \text{Tr} \left( R \wedge R \right) - \text{Tr} \left( F \wedge F \right) \right)$$

$$\frac{1}{4} \partial \mathcal{L} \quad \text{4 derivative}$$

which modify the no-go, permitting many (but not all) $\mathfrak{sl}_q$ vacua.

The resulting spaces are twisted manifolds $M$.

Finally, from $T$ calculate $\mathfrak{sl}_q$ the flow by

$$\mathfrak{sl}_q \quad H_3 = \frac{1}{\ell} \mathcal{J} \mathcal{J}.$$
In type II string theory, D-branes support gravitation and W3 couplings:

\[
W_3 = \frac{1}{k} \int_{\mathcal{M}} \left[ \frac{C}{\sqrt{A(R_T) / A(R_N)}} \right]^{2k+1} \sqrt{A(R_T) / A(R_N)}
\]

\[
\sqrt{A(R_T)} = 1 + \left( \frac{4\pi^2}{384\pi^2} \right) (r_T + r_N) + \ldots
\]

Argued from inflow. Similar couplings exist on D-particles.

In particular, D7 1/256 support

\[
\int C_4 \wedge \mathcal{P}, \quad \mathcal{P} = \frac{1}{8\pi^2} + \ldots
\]

This gives a source

\[
dF_5 = F_3 \wedge \mathcal{H}_2 + \psi \wedge \mathcal{E}_0
\]

\[
X_4 \text{ appearing in 4-dimensional superstring \textit{M}theory.}
\]

What about \textit{M}theory and IIA?

Note that all corrections that produce flux
include new higher derivative source of fine charge. Let's search for such potential sources.

II New Sources of Fermi Charge.

7-brane and metric are sufficient to induce charge in II B and violate the no-go time.

How about II A?

\[ m = C \gamma, \text{ could } 6 \text{-planes be different?} \]

The answer appears to be no. Recall that

\[ ds^2 = \sqrt{\left(dx^2 + c^2 d\omega^2 \right) + \sqrt{1 - (dy + A)^2}} \]

\[ \sqrt{V} = 1 + \frac{1}{2} \gamma, \quad y - y' + 2\pi \quad A \text{ at } d\omega \partial \phi \]

\[ ds^2 = d\theta^2 + \sin^2 \theta \ d\phi^2 \]

This is a perfectly regular metric.
For D-planes, the story looks similar.

\[ \text{orb} \to J_0(2N) \to \text{AH metric} \]

\[ \text{orb}^{-1} \to J_0(2N+1) \quad \text{massive DA} \]

\[ \text{orb}^+ \to J_0(N) \to \text{"frozen" O(4) sing.} \]

\[ \text{orb}^+ \to J_0(N) \quad \text{massive DA} \]

\[ \text{orb} \to J_0(N) \]

Each plane either lifts to a regular metric or has positive tension. These cannot evade the go-no go here.

In this respect DA and DB are very different.

Another way to see this is to note that

\[ \text{MS wrapped by orb / orb} \]

\[ \text{odd - dim.} \]

\[ \left( C \wedge A \right) \]

\[ \text{an - dim.} \]

These couplings induce no charge.
Let's learn something from duality. Imagine

\[ \int_{\mathcal{M}} \mathcal{F} = \frac{1}{8n} \int_{\mathcal{M}} \mathcal{F} = \mathcal{F} \]

Induced D7-brane charge.

Take \( y = \text{ constant} \) for \( x^1 \) corresponding to a winding direction. Define

\[ \mathcal{D}_6 \quad 0 \quad 1 \quad 2 \quad 4 \quad 5 \quad 6 \]

\[ \mathcal{D}_4 \quad 0 \quad 1 \quad 2 \quad 3 \quad 7 \]

As I'll explain, \( y \text{ constant} \) \( \mathcal{F} \) relates if there isn't an H-flux of the form

\[ H_{ij}: \text{This is a flux induced charge!} \]
Without any further work, we conclude that the way to evade the no-go’s is to consider a background metric and H-flux, and we these big must a odd - dimensional 4 derivative couplings.

Let me sketch the details: isometry direction is \( y \) \( dy = e^m e^m + \xi \xi \) \( \xi = f(x)/dy + \eta \)

The connection \( A \) encodes the topology of the \( S^1 \) bundle.

Then \( \hat{\omega} = \omega \wedge + \omega \wedge \eta \)

\( \hat{\omega} \eta = \hat{\omega} \eta \omega \) - form for full metric

\( \hat{\omega} \eta = \hat{\omega} \eta \omega \) - form depending on \( \xi \)

This allows us to integrate out the isometry direction.

Under Buscher: \( A_i dx^i \rightarrow \xi^i \frac{dx^i}{H} \)

Topology of \( \rightarrow H \)-flux \( S^1 \)-bundle
\[ T_0 - 3 \text{ unrun NV 5-done} \]

\[ \frac{1}{2} \left( \frac{1}{2} + \frac{1}{4} \right) x + 1 = \frac{1}{2} x \]

\[ V = \sqrt{\frac{1}{2} x + 1} \]

\[ \text{Returning to the general case:} \]

\[ \sum y \, dR \, n / \phi^2 = \sum y \, \phi^2 n + \pi \phi n / \phi^2 \]

\[ = x y \]

\[ x \phi^2 = \frac{1}{2} \frac{c}{\phi^2} \]

\[ \phi = \frac{1}{2} \frac{c}{\phi^2} \]

\[ H_3 = x^3 - 3 \phi \]

\[ \text{on all-branes} \]

\[ \sum (c_5) y \wedge x^3 \]

\[ \text{generally} \]

\[ \sum (c_5) \wedge x^3 \]

\[ \text{normal.} \]

\[ \text{We need a clean geometric understanding of these couplings! Physic from nearly inflation.} \]
One can lift this coupling to M-theory. The argument goes as follows:

**M-theory on T^7**

\[ \text{TIA 06-brane} \rightarrow \text{IIA closed string join for the 06-brane.} \]

\[ \int_{\Omega^2} C_3 \wedge \Omega^8 \text{ m5-brane charge} \]

\[ \int_{\Omega^{10}} C_3 \wedge \Omega^7 \]

\[ \text{Lifting to M-theory} \int C_6 \wedge X_5 \text{ induces M5 charge.} \]

\[ X_5 \text{ depends on G-flux!} \]

This is duality changing M-theory on M_8 with

\[ T^8 \rightarrow \text{M}_8 \text{ stacking w/} \int C_3 \wedge \Omega^2 \]

\[ \rightarrow \text{M}_8 \]
we don't need the detailed form of this condition, just the essence of the argument.

III Examples

Let's construct an M-theory example.

Take IIB $T^6 \times K3$. I choose $F_3$

$2 h (-1)^F \omega$

and $H_2$ flux that preserves either $N=1$ or $N=2$ SUSY.

Let me recall that $m$ on $T^6/Z_2 \simeq \frac{\text{II}B}{K3}$

(Dasgupta & Mullik and written)

Take an orbifold $K3$

$ds^2 = d\sigma^2 + d\rho^2 + d\sigma d\rho$

$\rho \in \mathbb{H}_{(1,1)}(K3, \mathbb{Z}) \in \mathbb{H}_{(1,1)}(K3, \mathbb{Z})$.

$H_2 = (2 + \rho) \wedge d\sigma + c.c.$

$F_3 = \iota(\rho - \overline{\rho}) \wedge d\sigma + c.c.$

So $\mathcal{S}_n = \frac{1}{2} (2 + \rho) + c.c.$
\( B' = \tilde{B}' + \tilde{A}' d\nu \quad C' = \tilde{C}' + \tilde{A}' d\nu \)

1. Analyze along \( \nu_1 \) and lift to M-theory.

\[
\begin{align*}
\delta_\alpha^\mu = & -\omega_{\mu
u} \eta_{\nu\rho} dx^\rho d\nu + \omega_{\nu} (d\nu d\nu + (d\nu)^2) + \omega_{\nu} ((d\nu + \tilde{A})^2 \\
+ & \omega_{\nu} ((d\nu + \tilde{A})^2)
\end{align*}
\]

\( \leftarrow \text{M-theory circle} \)

Flux:

\[
\begin{align*}
(\tilde{\omega}_\nu)^{ij} = & (\tilde{B}')^{ij} \\
(\tilde{\omega}_\nu)^{ij}_{\nu_1} = & (\tilde{C}')^{ij}_{\nu_1} \\
(\tilde{\omega}_\nu)^{ijkl} = & 3 (\tilde{C}')^{ijkl} \tilde{A}^k
\end{align*}
\]

\[
\begin{align*}
\sum_{\alpha} \int_{\alpha} & \to \chi_5 \\
\downarrow & \\
\frac{1}{15} / 26 & \nu
\end{align*}
\]

The manifold action yields:

\[
\begin{align*}
(\nu, \nu_1, \nu_0, y, \nu) \quad \to \quad (-\nu, -\nu_1, -\nu_0, -y, -\nu)
\end{align*}
\]

This is a Mink. flux cond. f

M-theory.