M-theory and Type IIA Flux Vacua

based on 1208.0261 w/ J. McOrist

Outline:

I Introduction

II New Sources of Brane Charge

III Examples

I Introduction

- Basic no-go thm forbidding flux compactifications in M-theory and string theory

Consider 11-dim SUGRA:

$$S_{11} = \frac{1}{2k^2}\int d^{11}x\,\sqrt{-g}\,\left(R - \frac{1}{2}|G|^2\right)$$
$$- \frac{1}{2k^2}\int \frac{1}{6}\, C\wedge G\wedge G$$

Take a warped product $M_4 \times_w M_7$

$$ds^2 = e^{2w(y)}\left(\hat{g}_{\mu\nu}\,dx^\mu dx^\nu + \hat{g}_{mn}(y)\,dy^m dy^n\right)$$

Lorentz $\Rightarrow$ $\underbrace{G_{mnpq}}_{\text{internal}}$, $G_{\mu\nu\rho\sigma} = f\sqrt{-g^{(4)}}\,\epsilon_{\mu\nu\rho\sigma}$

No sources $\Rightarrow$ $dG = 0$ and
$f = f_0\, e^{-4w}$

E.o.m. $R_{MN} = \frac{1}{12}\left(G_{MPQR}\,G_N{}^{PQR} - 2\,g_{MN}\,|G|^2\right)$

$$d * G = -\frac{1}{2}\, G\wedge G$$

Maximally symmetric space-time:

AdS_4 or Mink. requires $\hat{R}_{\mu\nu} = \hat{g}_{\mu\nu}\Lambda$

w/ Λ const.

$$R^{(4)} = -\frac{4}{3}f^2 - \frac{2}{3}|G^{int}|^2$$

(*) $$R^{(7)} = \frac{5}{6}|G^{int}|^2 + \frac{7}{6}|f|^2$$

} tracing over 4 or 7 dim.

$$\Rightarrow \quad R^{(4)} \leq 0$$
$$= e^{-2w}\left(\hat{R}^{(4)} - 4\hat{\nabla}^2 w - 36|\hat{\nabla} w|^2\right)$$
$$= 4e^{-2w}\left(\Lambda - \frac{1}{9}e^{-9w}\hat{\nabla}^2 e^{9w}\right)$$

$$\Lambda = \frac{\int_{M_7} e^{11w} R^{(4)}}{4\int_{M_7} e^{9w}}$$

So $\Lambda = 0 \Rightarrow R^{(4)} = 0 \Rightarrow$ all G-flux vanishes.

No Mink. or de Sitter.

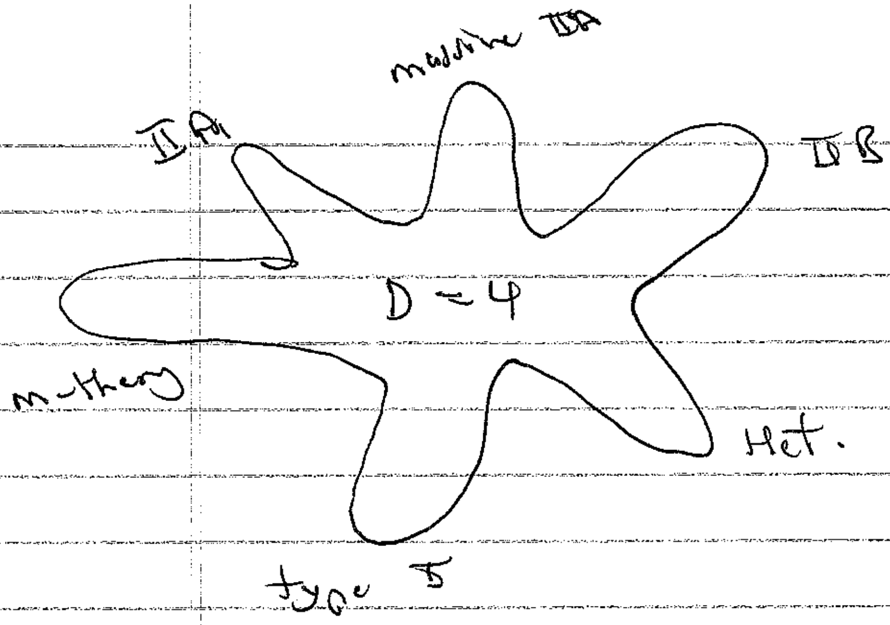
$$\hat{R}_{\mu\nu} = \hat{g}_{\mu\nu}\Lambda = \hat{g}_{\mu\nu}\Big(\hat{\nabla}^2 w + 9|\hat{\nabla} w|^2 - \frac{71}{144}f_0^2 e^{-6w} - \frac{1}{144}e^{2w}|G^{(int)}|^2\Big)$$

Strongest constraint since the r.h.s must be point-wise indep. of y.

Freund – Rubin AdS_4 solns correspond to ω const, $G^{int} = 0$. There are a few generalizations.

Generically Λ is o(curvature $R^{(7)}$). Flux solns w/ Λ parametrically smaller than $R^{(7)}$ encounter the Mink. no-go thm. None are known (to my knowledge).

Evading this no-go (Gibbons) requires higher derivative ingredients.



No mechanism is known in M-theory, IIA or massive IIA. I will comment on massive IIA later.

In Het / type I, there are R^2 couplings

$$\int \sqrt{g}\, R + R^2 + \ldots$$

$$dH = \frac{\alpha'}{4}\left(\mathrm{Tr}(R\wedge R) - \mathrm{Tr}(F \wedge F)\right)$$

4 derivative

which modify the no-go permitting Mink (but not AdS_4) solns.

The resulting spaces are torsional manifolds w/ a fund. form J correlated w/ the flux by SUSY $H_3 = i(\partial - \bar{\partial}) J$. Much progress here.

In type IIB string theory, D-branes support gravitational WZ couplings:

$$\int_{WZ}^{p+1} = \frac{T_p}{K}\int_{p+1} C \wedge e^{2\pi\alpha' F + B} \sqrt{\hat{A}(R_T)/\hat{A}(R_N)}$$

$$\sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}} = 1 + \frac{(4\pi^2\alpha')^2}{384\pi^2}\left(\mathrm{tr}\,R_T^2 - \mathrm{tr}\,R_N^2\right) + \ldots$$

Argued from inflow. Similar couplings exist on O-planes.

In particular D7 (O7)'s support

$$\int C_4 \wedge p_1 \qquad p_1 = \frac{1}{8\pi^2}\,\mathrm{tr}\,R\wedge R$$

This gives a source

$$dF_5 = F_3 \wedge H_3 + \sum_{\substack{D\\ \text{brane}\\ \text{sources}}} X_4 \wedge \delta_0^2$$

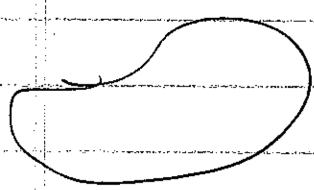
X_4 appropriate 4 derivative curvature interaction.

What about M-theory and IIA?

Note that all corrections that permit flux include new higher derivative sources of brane charge. Let's search for such potential sources.

II New Sources of Brane Charge.

D-branes and metric are sufficient to induce charge in IIB and violate the no-go thms. How about IIA?



$M = CY_3$. Could O6-planes be sufficient?

The answer appears to be no. Recall that

$$\text{D6-brane} \xrightarrow{\text{M-theory}} TN$$

$$ds^2 = V(dr^2 + r^2 d\Omega^2) + V^{-1}(dy \mp A)^2$$

$V = 1 + \frac{1}{r}$, $\quad y \sim y + 2\pi \quad A = \cos\theta \, d\phi$

$d\Omega^2 = d\theta^2 + \sin^2\theta \, d\phi^2$

This is a perfectly regular metric.

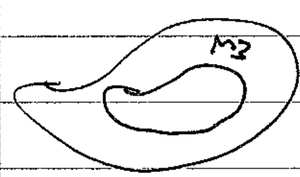
For O-planes, the story looks similar.

$O6^-$	$SO(2N)$	$\longrightarrow$	AH metric
$O6^{-'}$	$SO(2N+1)$		massive IIA
$O6^+$	$Sp(N)$	$\longrightarrow$	"frozen" D_4 sing.
$O6^{+'}$	$Sp(N)$		massive IIA
$D6$		$\longrightarrow$	TN

Each plane either lifts to a regular metric or has positive tension. These cannot evade the no-go thm.

In this respect IIA and IIB are very different.

Another way to see this is to note that



M_3 wrapped by $O6$ / $D6$

odd-dim.

$$\int C \wedge \underbrace{\hat{A}(\ )}_{\text{even-dim.}}$$

These couplings induce no charge.

Let's learn something from T-duality. Imagine

D7	0	1	2	3	4	5	6	7
D3	0	1	2	3	$\underbrace{\qquad\qquad\qquad}_{\mathrm{Tr}\, R \wedge R}$			

i.e. TN $\qquad \int_{TN} p_1 = \frac{1}{8\pi^2}\int_{TN} \mathrm{Tr}\, R\wedge R = 2$

Induced D3-brane charge.

Take y as a coord. for x^7 corresponding to a $U(1)$ isometry direction. Dualize.

D6	0	1	2	3	4	5	6	
D4	0	1	2	3				7

How? $\quad \int_{D7} C_4 \wedge X_4 \longrightarrow \int_{D6} (C_5)_{i_1 i_2 i_3 i_4 y}\, dx^{i_1}\ldots dx^{i_4}$

$\wedge\, (X_3)^y$

This must be the case and this must induce D4-brane charge.

As I'll explain in a moment $(X_3)^y$ vanishes if there isn't an H-flux of the form $H_{y i_1 i_2}$. This is a flux induced charge!

Without any further work, we conclude that the way to evade the no-go's is to consider a background metric and H-flux, and we these bizarre "odd-dimensional" 4 derivative couplings.

Let me sketch the details: isometry direction is y. $ds^2 = e^m e^m + e^y e^y$

$e^y = f(x)(dy + A)$

The connection A encodes the topology of the S^1-bundle.

Then $\hat{R} = d\hat{\omega} + \hat{\omega} \wedge \hat{\omega}$

2-form for full metric

$\hat{R}_{mn} = R_{mn}$ + forms depending on e^y

base

This allows us to integrate out the isometry direction.

Under Buscher: $A_i dx^i \longrightarrow B_{iy} dx^i$

topology of S^1-bundle $\longrightarrow$ H-flux ie: TN $\to$ smeared NV5-brane

$$ds^2 = V(dr^2 + r^2 d\Omega^2 + dy^2) \qquad \beta + y = \cos\theta$$

$$e^{2\phi} = V \qquad V = 1 + \frac{1}{r} \qquad d\Omega^2 = d\theta^2 + \sin^2\theta \, d\phi^2$$

Returning to the general case:

$$\int_y \mathrm{tr}\, R \wedge R = \int_y \hat{R}_{mn} \wedge \hat{R}_{nm} + \hat{R}_{yn} \wedge \hat{R}_{ny}$$

$$= X_3^y$$

$$X_3^y = -\underbrace{R_{mn}}_{2\text{-form}} \; \underbrace{d\, H_{mny}}_{1\text{-form}} + \ldots \; o(H) + o(H^3)$$

$H \to 0 \qquad X_3^y \to 0$

on D6-branes $\int (C_5)_y \wedge X_3^y$ or more generally $\int (C_5)_{ni} \wedge X_3^{ni}$

normal.

We need a clean geometric understanding of these couplings! Perhaps from anomaly inflow.

One can lift this coupling to M-theory. The argument goes as follows:

M-theory on TN

$\swarrow \qquad \searrow$

IIA D6-brane — IIA closed string soln for the D6-brane.

$\int C_3 \wedge X_8$ M2-brane charge (Becker & Becker)

$\swarrow \qquad \searrow$

$\int_{D6} C_3 \wedge P_1$ — $\int_{\mathbb{R}^{10}} C_3 \wedge X_7$

$\downarrow$

$\int_{D7} C_4 \wedge P_1 \longleftrightarrow \int_{IIB} C_4 \wedge X_6$ from considering an ALG space.

$\downarrow \qquad \downarrow$

$\int_{D6} (C_5)^n \wedge (X_3)^n$ — $\int_{IIA} C_5 \wedge X_5$

Lifting to M-theory $\int C_6 \wedge X_5$ induces M5 charge. X_5 depends on G-flux!

This is duality chasing M-theory on M_8 with

$$T^3 \rightarrow M_8 \quad \text{starting w/} \int C_3 \wedge X_8$$
$$\downarrow$$
$$M5$$

We don't need the detailed form of this coupling, just the essence of the argument.

III Examples

Let's construct an M-theory example.

Take IIB $\frac{T^2 \times K3}{\mathbb{Z}_2 \, (-1)^{F_L} \, \Omega}$. $\exists$ choices of F_3 and H_3 flux that preserve either $N=1$ or $N=2$ SUSY.

Let me recall that M on $T^5/\mathbb{Z}_2 \simeq$ IIB on K3

(Dasgupta & Mukhi and Witten)

Take an orbifold K3:

$$ds^2_{\text{orbifold}} = du\,d\bar{u} + dv\,d\bar{v} \qquad v = v_1 + i v_2$$

$$ds^2_{T^2} = dz\,d\bar{z}$$

$$\alpha \in H^{(1,1)}(K3, \mathbb{Z}) \qquad \beta \in H^{(2,0)}(K3, \mathbb{Z})$$

$$H_3 = (\alpha + \hat{\beta}) \wedge dz + \text{c.c.}$$

$$F_3 = i(\bar{\beta} - \alpha) \wedge dz + \text{c.c.}$$

$$\text{so} \quad \delta_2 = z\,(\alpha + \bar{\beta}) + \text{c.c.}$$

$$B_2 = \tilde{B}_2 + \tilde{A}\, dv_1 \qquad C_2 = \tilde{C}_2 + \tilde{C}_1\, dv_1$$

$\uparrow$ no dv_1

T-dualize along v_1 and lift to M-theory.

$$ds^2_M = e^{-w/2}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + e^{w}\left(dz\,d\bar{z} + du\,d\bar{u} + (dv_2)^2\right) + e^{-w/2}\,(dv_1 + \tilde{A})^2 + e^{-w/2}\,(dy + \tilde{C}_1)^2$$

$\uparrow$ M-theory circle

Flux

$$(\hat{C}_3)_{ijy} = (\tilde{B}_2)_{ij}$$
$$(\hat{C}_3)_{ijv_1} = (\tilde{C}_2)_{ijv_1}$$
$$(\hat{C}_3)_{ijk} = 3\,(\tilde{C}_2)_{[ij}\,\tilde{A}_{k]}$$

Space $\quad T^2 \longrightarrow X_7$

$$\downarrow$$

$$T^5/\mathbb{Z}_2$$

The orientifold action sends:

$$(u, v_1, v_2, y, C_3) \longrightarrow (-u, -v_1, -v_2, -y, -C_3)$$

This is a Mink. flux compact. of M-theory.