Black holes and fuzzballs

Oleg Lunin

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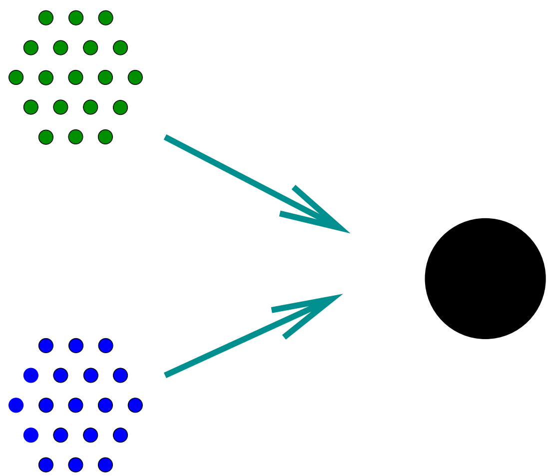
OL–Mathur–Turton, 1208.1770
Giusto–OL–Mathur–Turton, 1211.0306
OL–Mathur, 1211.5830

Black hole information paradox

- Black holes and Hawking radiation
  - classical theory: no-hair theorem



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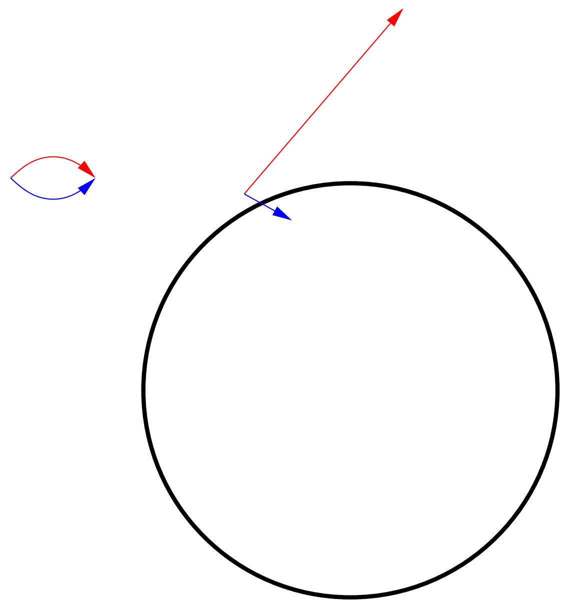

Hawking '74

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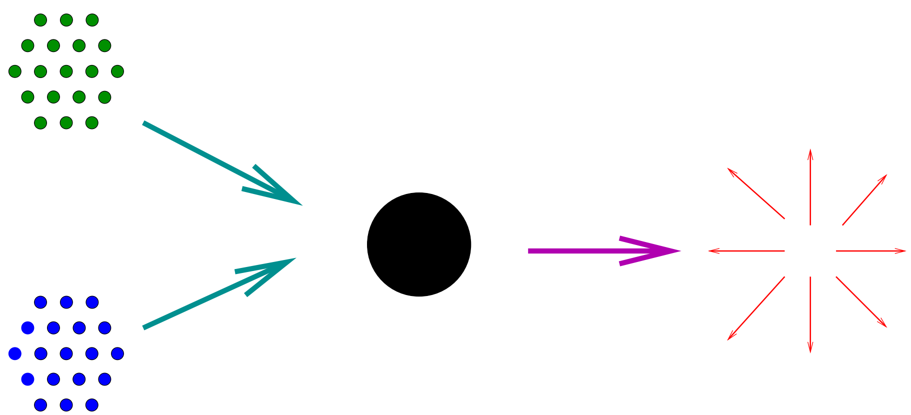

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  - large number of microstates: $S = \frac{A}{4}$
  - paradox: pure state $\Rightarrow$ mixed state

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- Options for resolving the paradox
  - information is lost
  - loophole in Hawking's argument

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- Information paradox and AdS/CFT
  - evolution is unitary
  - paradox is not eliminated

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  - D1–D5–P states localized at the “horizon"
  - momentum modes on the torus
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Black holes in AdS/CFT

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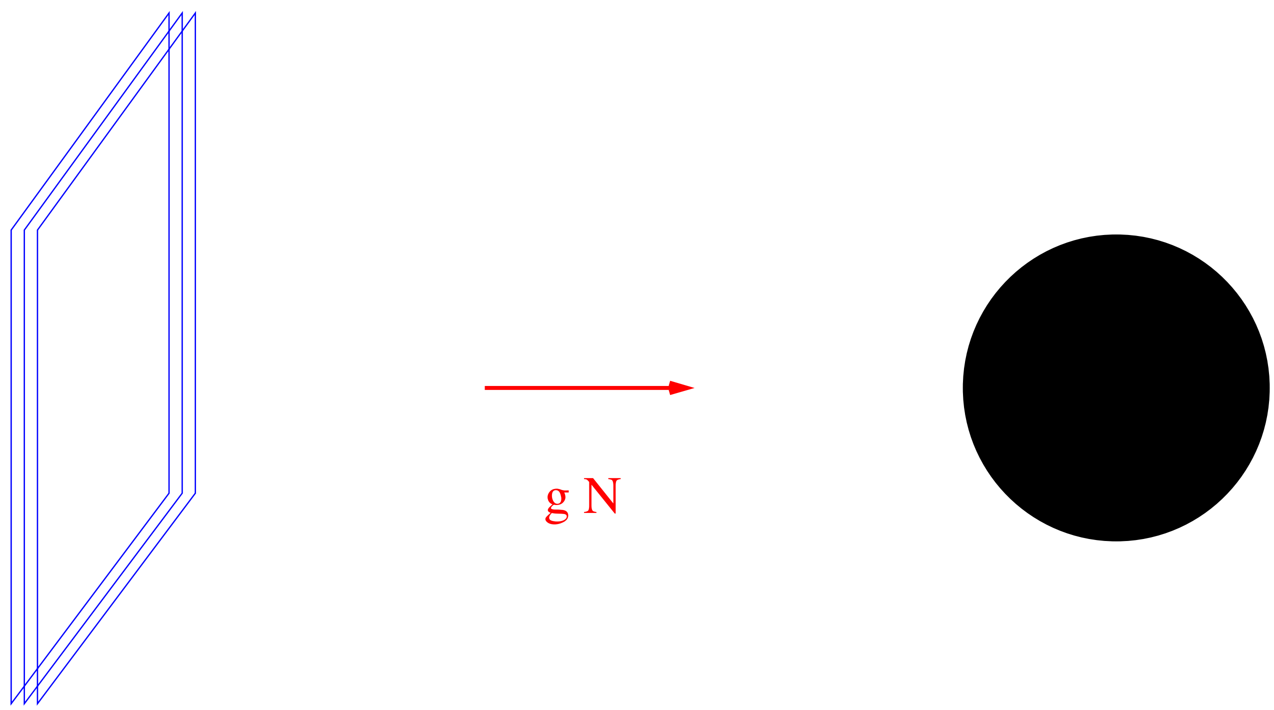

Maldacena '97

Black holes in AdS/CFT

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  - unitary QFT $\Rightarrow$ option B
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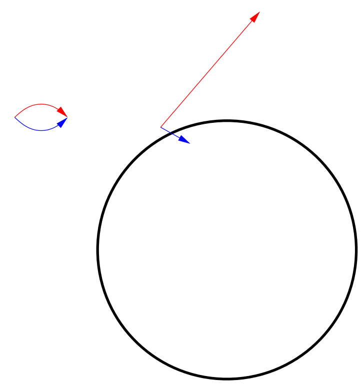
- Geometry must change at the horizon



Entropy and microscopic states

Entropy and microscopic states

- D1–D5–P system

	1	2	3	4	5	6	7	8	9
D1					$\bullet$	$\sim$	$\sim$	$\sim$	$\sim$
D5					$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
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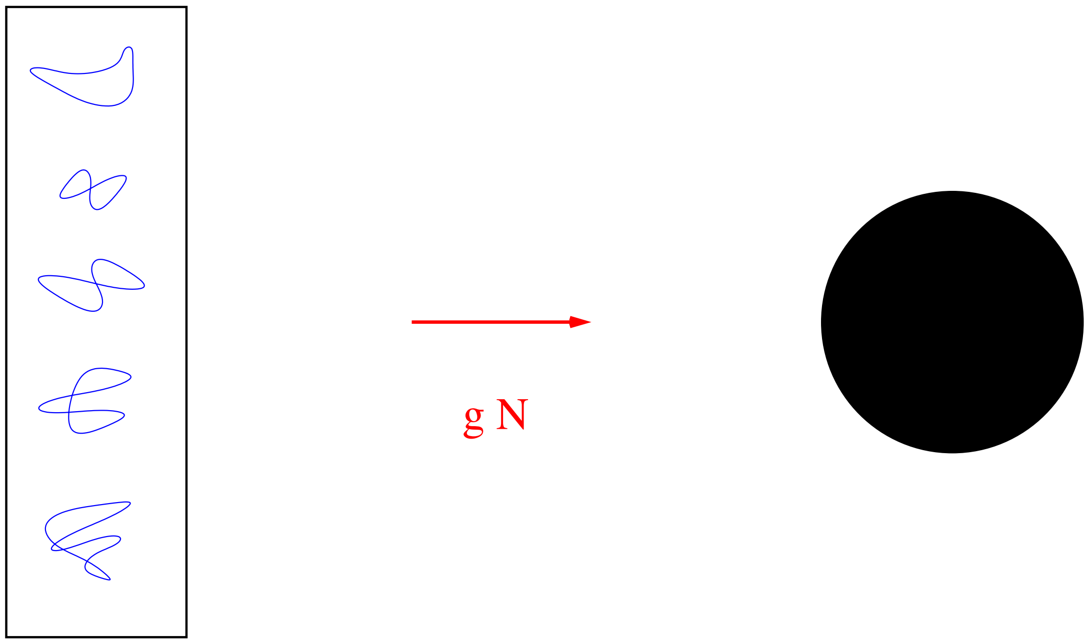
- Number of states at weak coupling:

$$S = k \ln \Gamma = \frac{A}{4\hbar G_N}$$

Strominger–Vafa '96

Entropy and microscopic states

- State counting



Strominger–Vafa '96

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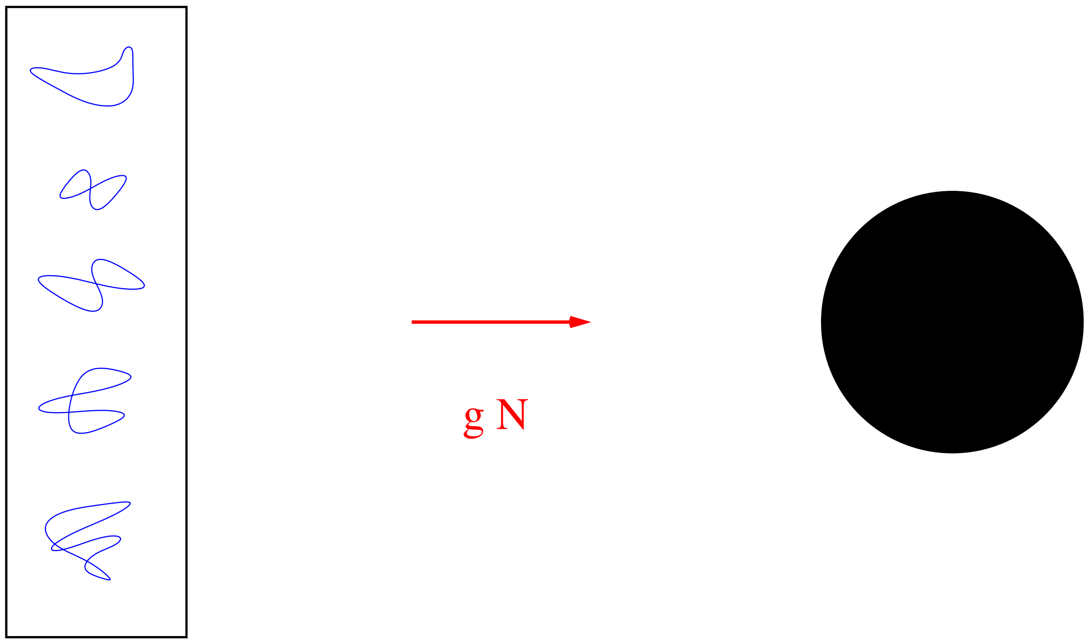
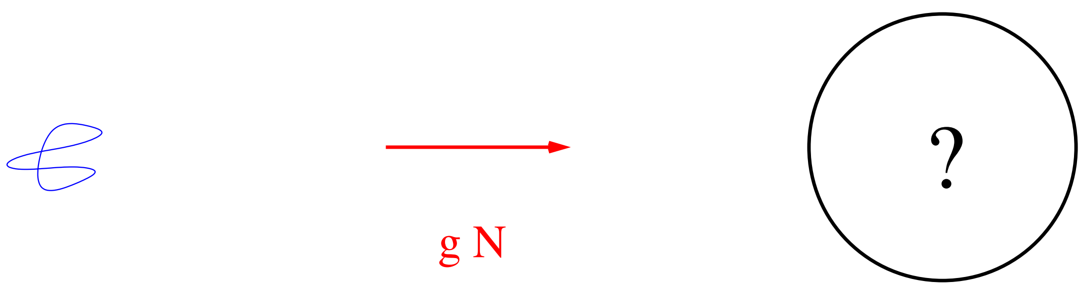


Strominger–Vafa '96

- Individual states



Fuzzballs

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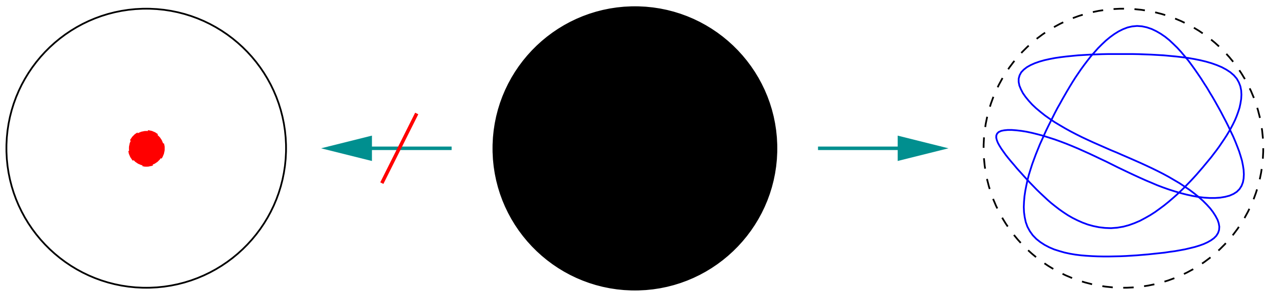

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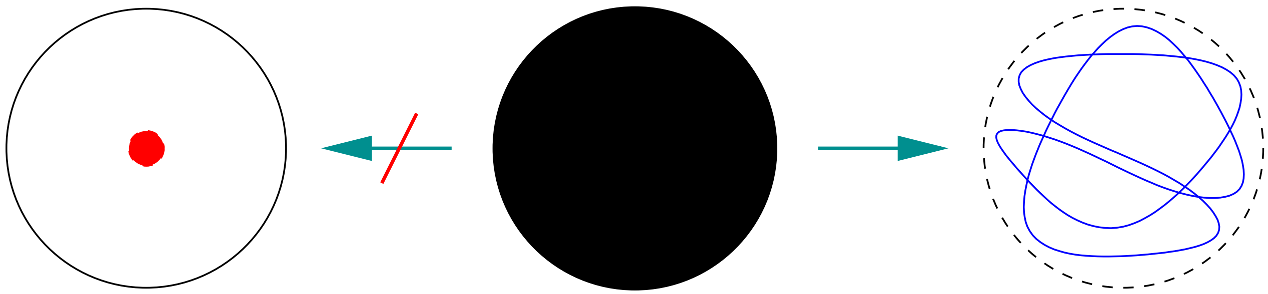

  - individual horizon–free metric for every state
  - distinctive features extending to the horizon
  - corrections to Hawking radiation

Lunin–Mathur ’01-02; Mathur ’04

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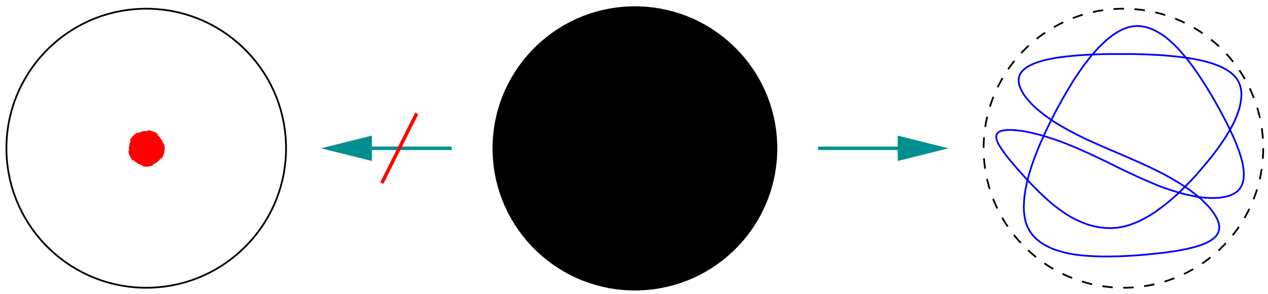

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Lunin–Mathur '01-02; Mathur '04

semiclassical state	$\Leftrightarrow$	solution to SUGRA
generic state	$\Leftrightarrow$	stringy geometry

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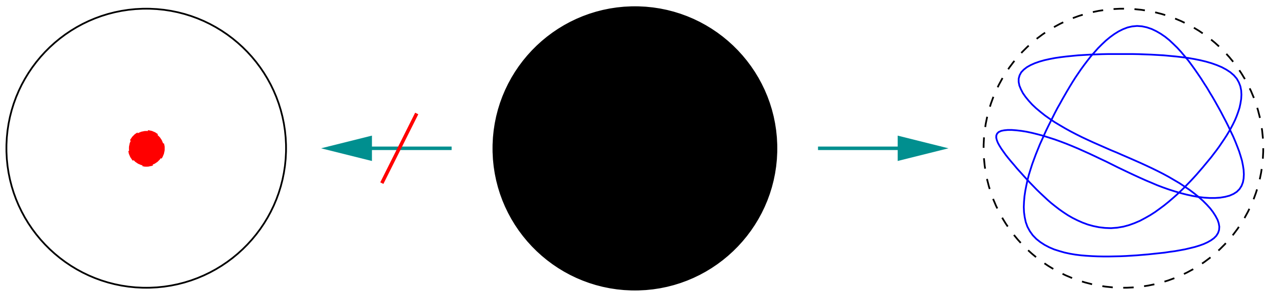

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- Horizon – coarse graining over geometries

Fuzzballs for D1–D5 system

- Microstates and regular geometries

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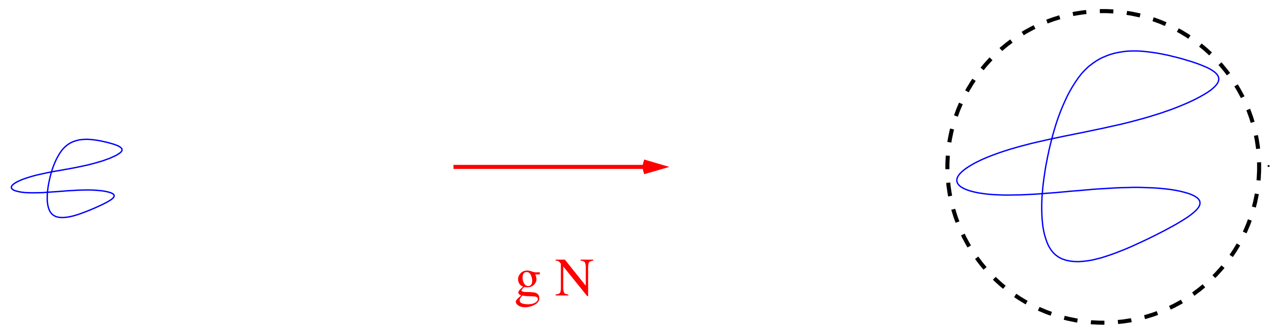

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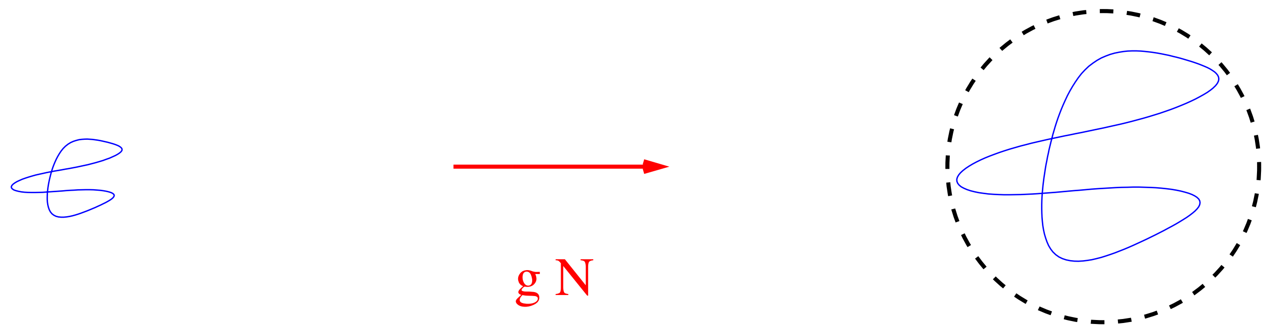

vibrating string: $\mathbf{F}(v)$ $\Leftrightarrow$ geometry with

$$H_5 = 1 + \int \frac{dv}{(\mathbf{x}-\mathbf{F})^2}\ , \ \ldots$$

OL–Mathur '01

vibration in 10D $\Leftrightarrow$ regular solution of SUGRA

OL–Maldacena–Maoz '02

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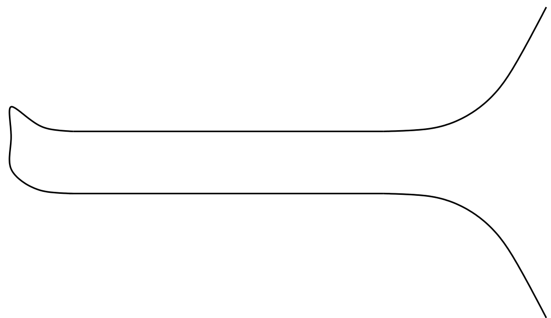

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$r,\quad S^3$

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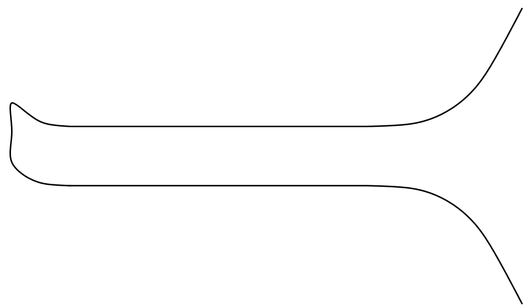

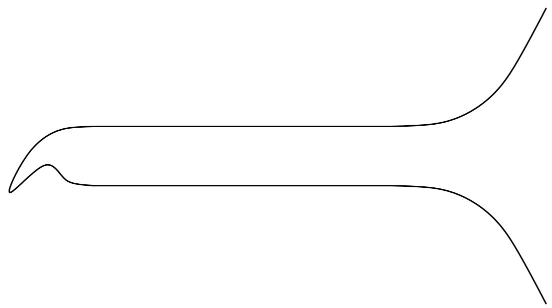

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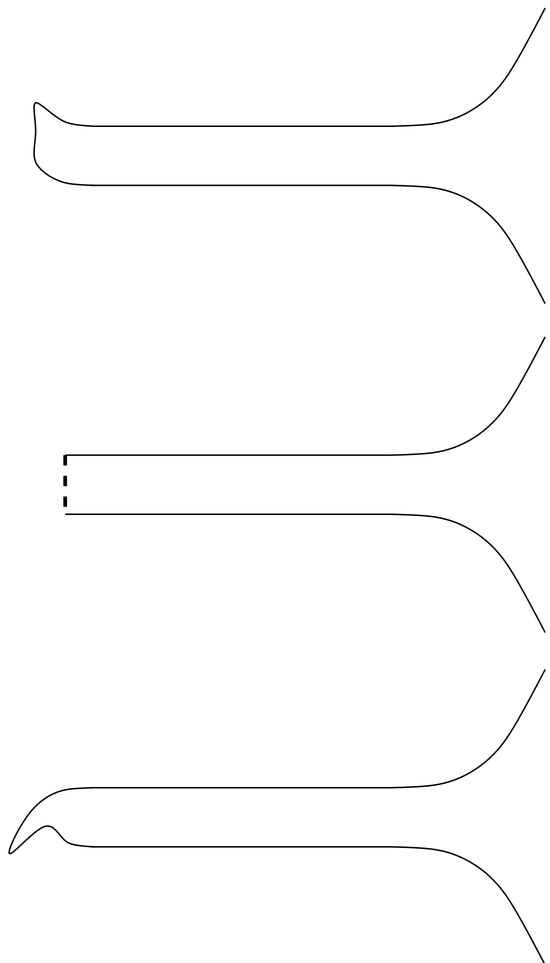

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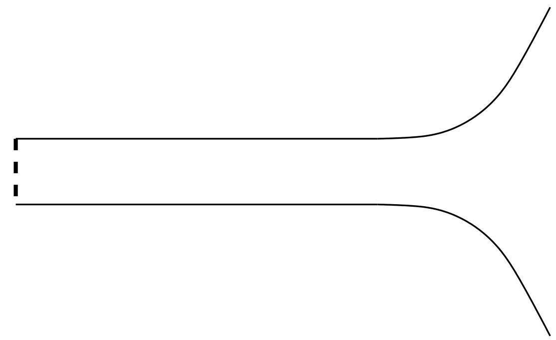

- Horizon as coarse graining
  - D1–D5: stretched horizon with Planckian area

    Sen '95
  - size of the fuzzball $\sim$ size of the horizon

    Lunin–Mathur '02

D1–D5–P and spectral flow

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  - boundary conditions for fermions

$$G(z) = -e^{i\pi\eta} G(e^{2\pi i} z), \quad J^{\pm}(z) = e^{\mp 2\pi i\eta} J^{\pm}(e^{2\pi i} z)$$

Schwimmer–Seiberg '87

D1–D5–P and spectral flow

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- Momentum and spectral flow in $N = (4,4)$ SCFT$_2$
  - spectral flow in CFT $\Leftrightarrow$ diff in AdS
  - mixture between AdS_3 and S^3 $(\psi \to \psi + \eta(t-y), \ldots)$

$$ds^2 = L^2 \left[-(r^2+1)dt^2 + \frac{dr^2}{r^2+1} + r^2 dy^2 + d\theta^2 + \cos^2\theta d\psi^2 + \sin^2\theta d\phi^2 \right]$$

  - momentum charge is generated
- Integer spectral flow is a regular diffeomorphism

D1–D5–P and spectral flow

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- Adding momentum to D1–D5 system
  - remove flat asymptotics
  - perform spectral flow
  - attach flat asymptotics

OL '04; Giusto–Mathur–Saxena '04

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- More states of D1–D5–P system
  - general solution in terms of 4D hyper–Kähler base

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  - microscopic interpretation?

Fractional spectral flow

- CFT dual of the D1–D5 system
  - marginal deformation of the free CFT on orbifold
  - orbifold point: $(T^4)^N/S_N$, $N = n_1 n_5$

de Boer '98; Seiberg–Witten '99; . . . .

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$$X_1 \to X_2 \to \ldots X_k \to X_1$$

  - functions are single–valued on the covering space
  - microstate with $\sigma_k \Rightarrow$ SF on the cover $\Rightarrow$ FSF

$$\psi(z) = -e^{i\pi\eta}\psi(e^{2\pi ik}z)$$

  - gravity: $[\sigma_k]^{N/k} \Rightarrow$ conical singularity

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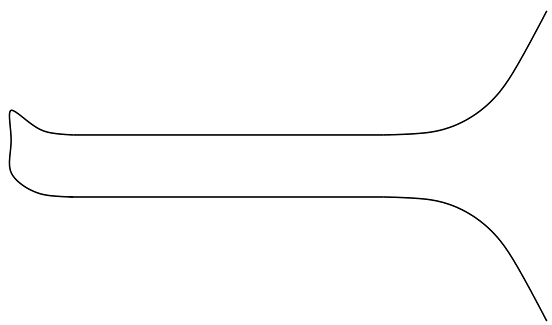
- Fractional spectral flow in gravity
  - AdS region:

$$\psi \to \psi + \frac{s}{k}(t-y), \qquad \phi \to \phi - \frac{s}{k}(t-y)$$

  - flat asymptotics – GMR or limit of a black hole
  - conical singularity associated with σ_k softens

Properties of “fractional microstates"

- Geometry is modified in the “cap"



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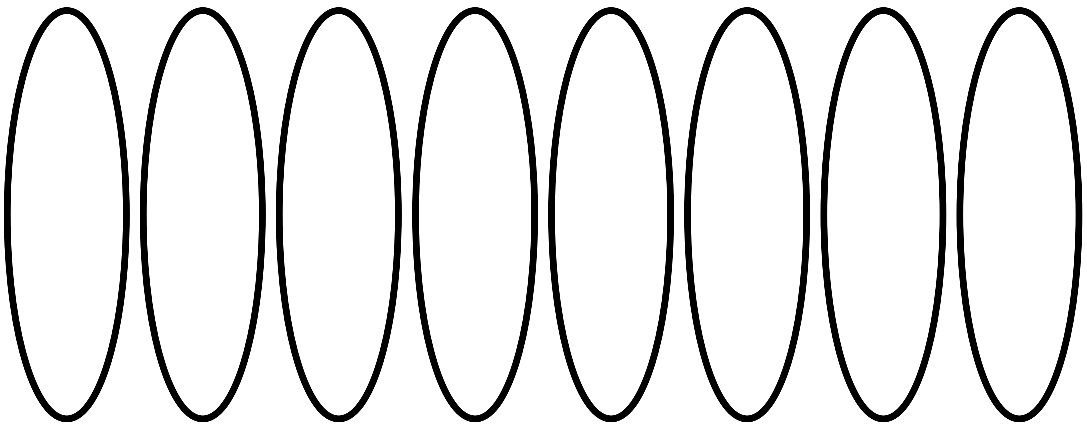

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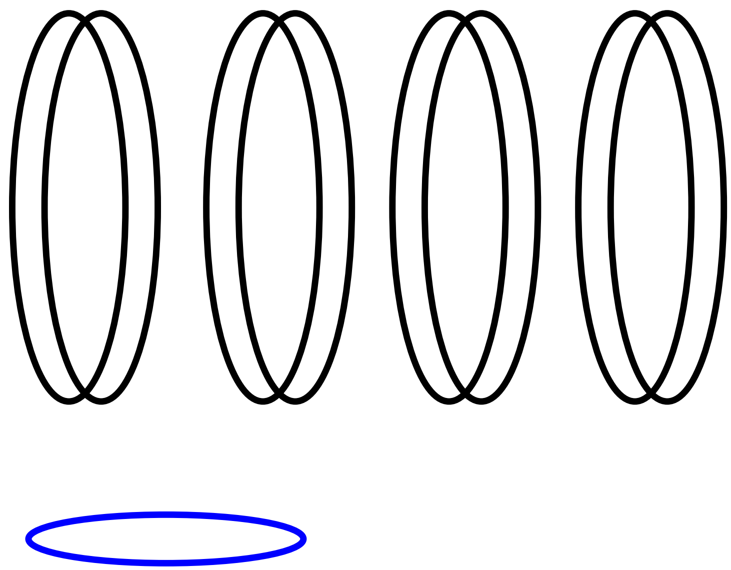

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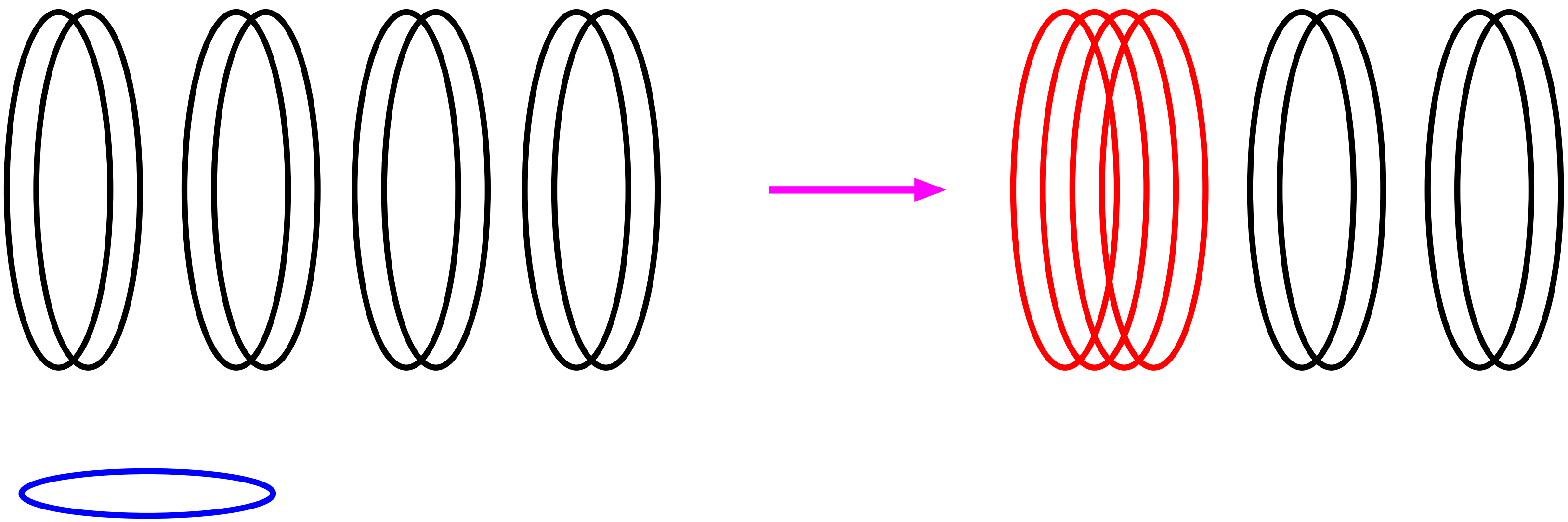

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- Contributions to the energy
  - fermions on a longer string $\Rightarrow \Delta h_f = -\frac{ls}{k}$
  - bosons from the vertex operator $\Rightarrow \Delta h_b = \frac{1}{k}$
  - ang. mom. from the vertex operator: $\Delta h_J = \frac{m(2s+1)}{k}$
  - momentum of the excitation: $n_p = \bar{h} - h$

Giusto–OL–Mathur–Turton ’12

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Giusto–OL–Mathur–Turton '12

$$\omega = \frac{l + 2 + 2(2s+1)m}{k} - n_p$$

Excitations on the torus

- Adding momentum to D1–D5
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  - start with known microstate
  - solve for linear perturbation
  - guess the solution
  - check the eqns for Killing spinors

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- Deformation of GMR: OL–Mathur–Turton '12

$$
\begin{aligned}
ds^2 &= -e^{\hat{v}} e^{\hat{u}} + H ds_4^2 + dz^\alpha dz^\alpha \,, \\
C^{(2)} &= \frac{1}{2} e^{\hat{v}} \wedge e^{\hat{u}} + e^{\hat{v}} \wedge \mathcal{A} + \sigma_2 \,, \\
e^{\hat{v}} &= H^{-1}(dv + \beta) \,, \quad e^{\hat{u}} = du + \omega + \frac{FH}{2} e^{\hat{v}} + \Phi_\alpha(v, x) dz^\alpha \,.
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  - linear equation for the deformation

$$d\left(*_6\left[D\Phi_\alpha+\partial_v\Phi_\alpha H e^{\hat{v}}\right]\right)=0$$

  - regular GMR geometry $\Rightarrow$ regular deformation
  - CFT excitation on the boundary $\Rightarrow$ unique solution

Relation to other constructions

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    Horowitz–Marolf '96
  - GV is extended to some non–static geometries

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- Pp–wave on D1–D5–P via dualities

$$
\begin{aligned}
ds^2 &= -e_0^{\hat{v}} e_0^{\hat{u}} - \frac{1}{4}\Phi_\alpha \Phi_\alpha (e_0^{\hat{v}})^2 + H ds_4^2 + dz^\alpha dz^\alpha \,, \\
B &= -\frac{1}{2}\Phi_\alpha e_0^{\hat{v}} \wedge dz^\alpha, \qquad C^{(4)} = \frac{1}{24}\left[e_0^{\hat{v}} \wedge \varepsilon_{\alpha\beta\gamma\delta}\Phi_\alpha dz^\beta \wedge dz^\gamma \wedge dz^\delta + dual\right] , \\
C^{(6)} &= \left[\frac{1}{2} e_0^{\hat{v}} \wedge e_0^{\hat{u}} + e_0^{\hat{v}} \wedge \mathcal{A} + \sigma_2\right] dz_1 \wedge dz_2 \wedge dz_3 \wedge dz_4 \,.
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$$

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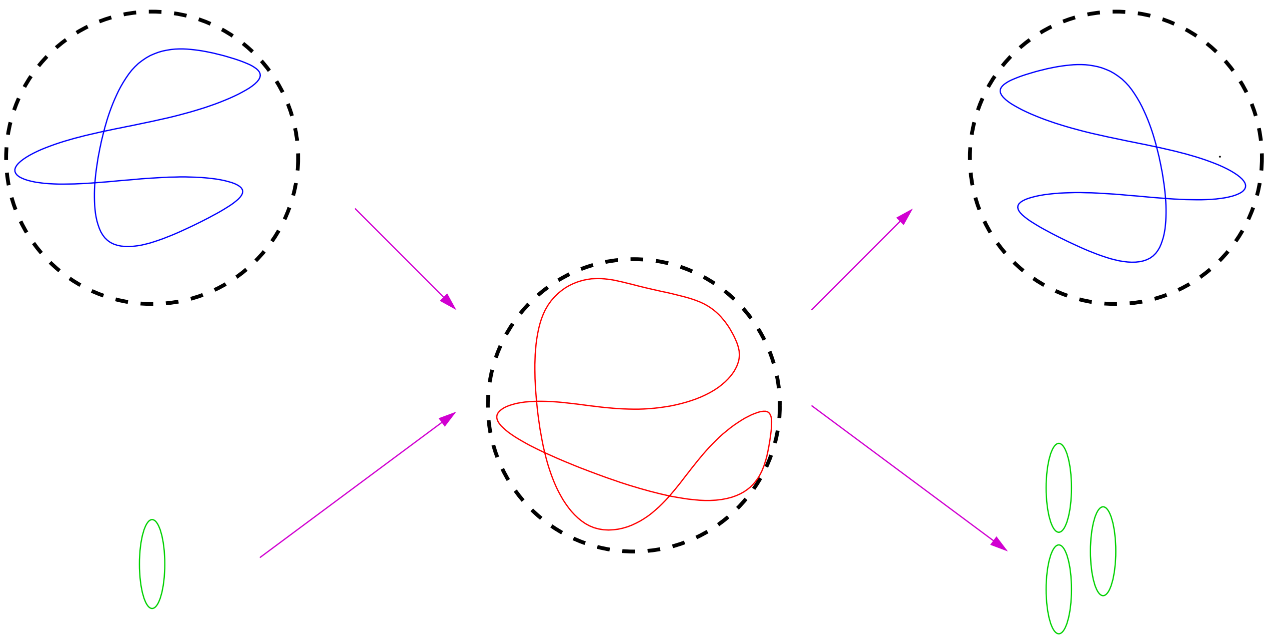

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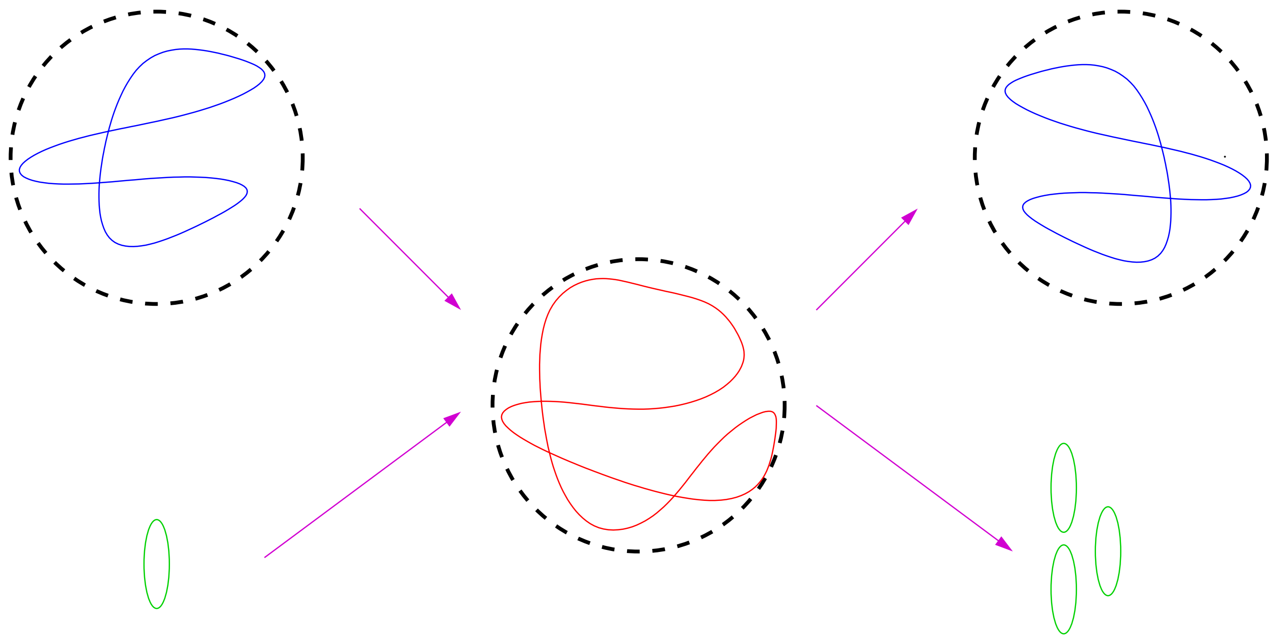

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Lunin–Mathur ’12

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