On the completion of the one-loop R^4 coupling in type II string theory

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26 April 2013

JTL and R. Minasian, arXiv:1304.3137

The low energy limit of string theory

- In the classical limit, string theory reduces to an effective theory of gravity + supersymmetry
  - For the type IIA string

$$\begin{aligned}\mathcal{L} &= e^{-2\phi}\left[R*1+4d\phi\wedge*d\phi-\tfrac{1}{2}H\wedge*H\right]\\ &\quad -\tfrac{1}{2}F_2\wedge*F_2-\tfrac{1}{2}F_4\wedge*F_4-\tfrac{1}{2}B_2\wedge F_4\wedge F_4\end{aligned}$$

  - Includes both NSNS and RR fields

$$\begin{aligned}\text{NSNS:}&\qquad g_{\mu\nu},\ B_{\mu\nu},\ \phi\\ \text{RR:}&\qquad F_2,\ F_4\end{aligned}$$

- In many applications, this is all we need
  - Classical supergravity solutions
  - Supersymmetric backgrounds, string compactification
  - AdS/CFT in the large N, large λ limit

Beyond the low energy limit

- Two categories of stringy corrections
  1. String loop corrections with $g_s \sim e^{\phi}$
  2. String α' corrections (higher derivative corrections)
- Distinguishes string theory from supergravity

$$\mathcal{L} = e^{-2\phi}\mathcal{L}_0 + \mathcal{L}_1 + e^{2\phi}\mathcal{L}_2 + \cdots$$

$$\mathcal{L}_g \sim \sum_{n\geq 1} (\alpha')^{n-1}(2n \text{ derivative terms})$$

- In AdS/CFT

$$g_s \to \frac{\lambda}{N}, \qquad \alpha' \to \frac{1}{\sqrt{\lambda}}$$

so stringy corrections give us information about finite coupling and finite N effects

Higher curvature corrections in string theory

- How do we determine the stringy corrections?
  - Reconstruct an effective action from string scattering amplitude calculations
  - For α' corrections, perform sigma-model loop calculations
  - Construct supersymmetric higher derivative invariants
  - Anomalies, consistency, symmetries, etc.
- For the type II string, the first correction arises at the level of the four-point function

$$\mathcal{A}_4 \sim h^{(1)} h^{(2)} h^{(3)} h^{(4)}$$

  - But the result must be gauge invariant

$$h \quad \to \quad k_{[\mu} k^{[\rho} h_{\nu]}{}^{\sigma]} \sim R_{\mu\nu}{}^{\rho\sigma}$$

  - So the four-point function gives an $\alpha'^3 R^4$ correction to the effective action (at tree-level and one-loop)

R^4 corrections in type II string theory

- For the type II string at $\mathcal{O}(\alpha'^3)$

$$e^{-1}\mathcal{L} \sim e^{-2\phi}(t_8 t_8 R^4 + \tfrac{1}{8}\epsilon_{10}\epsilon_{10}R^4) + (t_8 t_8 R^4 \mp \tfrac{1}{8}\epsilon_{10}\epsilon_{10}R^4) + (t_8\epsilon_{10} \pm \epsilon_{10} t_8) B R^4$$

  – At one loop in the RNS formalism

$$\begin{array}{ccc} \text{even spin structure} & \rightarrow & t_8 \\ \text{odd spin structure} & \rightarrow & \epsilon_{10} \end{array}$$

- The CP-odd term vanishes for IIB
  – For IIA it has the familiar expression

$$B \wedge X_8(R) = \frac{1}{192(2\pi)^4} B \wedge [\mathrm{Tr}\, R^4 - \tfrac{1}{4}(\mathrm{Tr}\, R^2)^2]$$

  – Related to the five-brane anomaly

Completing the R^4 couplings

- The R^4 terms as written are necessarily incomplete
  - The full supergravity multiplet must show up

Part of a higher derivative supersymmetry invariant

  - What about T-duality?

$$g_{\mu\nu} \quad \leftrightarrow \quad B_{\mu\nu}$$

- We focus on the NSNS sector only: $(g_{\mu\nu}, \quad B_{\mu\nu}, \quad \phi)$
  - Then, at the two-derivative level

$$R \sim \nabla H \sim H^2 \sim \nabla\nabla\phi \sim (\nabla\phi)^2$$

- so, for example

$$R^4 \sim R^2(\nabla H)^2 \sim H^2 R^3 \sim \cdots \sim H^8$$

Introduce the connection with torsion

- How we compute the closed string four-point function

$$V^{(-1,-1)}(k,\theta) = \theta_{\mu\nu}\delta(\gamma)\delta(\tilde{\gamma})\psi^{\mu}\tilde{\psi}^{\nu}e^{ik\cdot X}$$

where the polarization tensor is

$$\theta_{\mu\nu} = h_{\mu\nu} + b_{\mu\nu} + \tfrac{1}{2}(\eta_{\mu\nu} - \bar{k}_{\mu}k_{\nu} - k_{\mu}\bar{k}_{\nu})\phi$$

- Captures the scattering of any combination of NSNS fields
- Instead of $k_{[\mu}k^{[\rho}h_{\nu]}{}^{\sigma]} \sim R_{\mu\nu}{}^{\rho\sigma}$, we have

$$k_{[\mu}k^{[\rho}\theta_{\nu]}{}^{\sigma]} \sim R_{\mu\nu}{}^{\rho\sigma} + e^{-\phi/2}\nabla_{[\mu}H_{\nu]}{}^{\rho\sigma} - \delta_{[\mu}{}^{[\rho}\nabla_{\nu]}\nabla^{\sigma]}\phi$$

  – Simpler when Weyl scaled to the string frame

$$\rightarrow \qquad R_{\mu\nu}{}^{\rho\sigma} + \nabla_{[\mu}H_{\nu]}{}^{\rho\sigma}$$

- This is the linearized curvature computed from a connection with torsion

$$R(\Omega_{+}) \qquad \text{where} \qquad \Omega_{\pm} = \omega \pm \tfrac{1}{2}\mathcal{H}$$

What we expect

- At the linearized level $R \to R(\Omega_+)$ is the correct thing to do
  - The one loop couplings receive contributions from the difference spin structure sectors

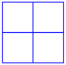
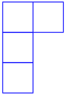
	Without B	With B
CP-even		
e-e	$t_8 t_8 R^4$	$t_8 t_8 R(\Omega_+)^4$
o-o	$\epsilon_{10}\epsilon_{10} R^4$	$\epsilon_{10}\epsilon_{10} R(\Omega_+)^4$
CP-odd		
e-o & o-e	$B \wedge X_8(\omega)$	$\frac{1}{2} B \wedge [X_8(\Omega_+) + X_8(\Omega_-)]$

D. J. Gross and J. H. Sloan, Nucl. Phys. B 291, 41 (1987)

- But is this complete (in the NSNS sector) at the non-linear level?

But wait . . . there's more

- Use of the connection with torsion is compatible with T-duality
- However, there are hints that this is incomplete
  - Different symmetry properties

$R_{\mu\nu\rho\sigma}$:  vs. $\nabla_\mu H_{\nu\rho\sigma}$: 

  - String three-point function computed for type II on K3

e-e: $R^2_{\mu\nu\rho\sigma} + \frac{1}{2}\nabla_\mu H_\nu{}^{ab}\nabla_\nu H_\mu{}^{ab} \quad \rightarrow \quad t_4 t_4 R(\Omega_+)^2$

o-o: $R^2_{\mu\nu\rho\sigma} + \frac{1}{2}\nabla_\mu H_\nu{}^{ab}\nabla_\nu H_\mu{}^{ab} \quad \rightarrow \quad -\frac{1}{8}\epsilon_6\epsilon_6 R(\Omega_+)^2 + H_{\mu\rho}{}^a H_{\nu\sigma}{}^a R^{\mu\nu\rho\sigma}$

A. Gregori, et al., Nucl. Phys. B 510, 423 (1998)

- We conjecture

$R \rightarrow R(\Omega_+)$ complete except in the o-o sector

Evidence supporting this conjecture

- Beyond the linearized Riemann contribution

$$R(\Omega_\pm)_{\mu\nu}{}^{\alpha\beta} = R_{\mu\nu}{}^{\alpha\beta} \pm \nabla_{[\mu} H_{\nu]}{}^{\alpha\beta} + \tfrac{1}{2} H_{[\mu}{}^{\alpha\gamma} H_{\nu]\gamma}{}^{\beta}$$

so

$$R(\Omega_+)^4 \sim R^4 + \cdots + H^8 \qquad \text{(automatically even in } H\text{)}$$

- Conjecture: this is all that is needed in the e-e sector and the CP-odd sector
- Some evidence
  - T-duality in the CP-odd sector
  - Heterotic/IIA duality
  - String four-point function for type II on K3 in the o-o sector

A note on the CP-odd sector

- The $B \wedge X_8$ term is related to anomalies, and is highly constrained
- Consider the shift $B \wedge X_8 \;\rightarrow\; B \wedge \tilde{X}_8 = B \wedge (X_8 + Y)$
  - Invariance under B field gauge transformations: $dY = 0$
  - NS5-brane anomaly cancellation: Y should be exact
- The replacement $\omega \rightarrow \omega + \frac{1}{2}\mathcal{H}$ automatically satisfies both conditions
- For example, take

$$\begin{aligned} 8\pi^2 X_4 &= \operatorname{Tr} R(\Omega_+) \wedge R(\Omega_+) \\ &= \operatorname{Tr} R \wedge R + d[\operatorname{Tr}(R \wedge \mathcal{H} + \tfrac{1}{4}\mathcal{H} \wedge d\mathcal{H} + \tfrac{1}{12}\mathcal{H}^3)] \end{aligned}$$

T-duality in the CP-odd sector

- Consider a circle reduction at constant radius

$$ds^2 = \eta_{\alpha\beta} e^\alpha e^\beta + e^9 e^9, \qquad de^9 = T$$

$$B_2 = b_2 + b_1 \wedge e^9, \quad H_3 = h_3 + \tilde{T} \wedge e^9, \qquad db_1 = \tilde{T}$$

- Let $X_4 = \tilde{X}_4 + \tilde{X}_3 \wedge e^9$
- Then

$$\int_{\mathcal{M}_6} B \wedge X_4(\Omega_+) = \int_{\mathcal{M}_5} b_1 \wedge (\tilde{X}_4 - \tilde{X}_2 \wedge T) - h_3 \wedge \tilde{X}_2$$

where

$$\begin{aligned} -8\pi^2(\tilde{X}_4 - \tilde{X}_2 \wedge T) &= R^{\alpha\beta}(\omega_+) \wedge R^{\alpha\beta}(\omega_+) \\ &\quad -\tfrac{1}{2} R^{\alpha\beta}(\omega_+) \wedge T_{+\,\gamma}^{\alpha} e^\gamma \wedge T_{+\,\delta}^{\beta} e^\delta \\ &\quad +\tfrac{1}{8} D_\alpha(\omega_-) T_+ \wedge D^\alpha(\omega_-) T_+ \end{aligned}$$

- This expression is T-duality invariant

Heterotic/IIA duality

- Consider the duality between Heterotic on T^4 and IIA on $K3$
- The tree-level R^2 correction is known

$$e^{-1}\mathcal{L}^{\text{het}}_{d=6} = e^{-2\phi}[R + 4\partial\phi^2 - \tfrac{1}{12}H^2_{\mu\nu\rho} + \tfrac{1}{8}\alpha' R_{\mu\nu\rho\sigma}(\Omega_+)^2 + \cdots]$$

$$dH = \tfrac{1}{4}\alpha' \text{Tr}\, R(\Omega_+) \wedge R(\Omega_+)$$

- So all we need to do is dualize

$$e^{-2\phi}H^{\text{het}} = *H^{\text{IIA}}, \qquad e^{-2\phi}g^{\text{het}}_{\mu\nu} = g^{\text{IIA}}_{\mu\nu}, \qquad \phi = -\varphi^{\text{IIA}}$$

(We find that no correction is needed at $\mathcal{O}(\alpha')$)

- Note that the connection with torsion does not map directly

$$\Omega^{\text{het}}_+ \quad \not\to \quad \Omega^{\text{IIA}}_+$$

The resulting IIA Lagrangian

- In the IIA frame ($dH = 0$), we find

$$
\begin{aligned}
e^{-1}\mathcal{L}^{\mathrm{IIA}}_{d=6} &= e^{-2\varphi}[R + 4\partial\varphi^2 - \tfrac{1}{12}H^2_{\mu\nu\rho}] \\
&\quad + \tfrac{1}{16}\alpha'\left[t_4 t_4 R(\Omega_+)^2 - \tfrac{1}{8}\epsilon_6\epsilon_6\left(\boxed{R(\Omega_+)^2 + \tfrac{4}{3}H^2 R(\Omega_+) + \tfrac{1}{9}H^4}\right)\right] \\
&\quad - \tfrac{1}{8}\alpha' B \wedge \left[\mathrm{Tr}\, R(\Omega_+)\wedge R(\Omega_+) + \mathrm{Tr}\, R(\Omega_-)\wedge R(\Omega_-)\right]
\end{aligned}
$$

where

$$
\begin{aligned}
t_4 t_4 R(\Omega_+)^2 &= R_{\mu\nu\rho\sigma}(\Omega_+)R^{\mu\nu\rho\sigma}(\Omega_+) \\
\epsilon_6\epsilon_6 R(\Omega_+)^2 &= \epsilon_{\alpha\beta\mu_1\cdots\mu_4}\epsilon^{\alpha\beta\nu_1\cdots\nu_4}R^{\mu_1\mu_2}{}_{\nu_1\nu_2}(\Omega_+)R^{\mu_3\mu_4}{}_{\nu_3\nu_4}(\Omega_+) \\
&= -8(R_{\mu\nu\rho\sigma}(\Omega_+)R^{\rho\sigma\mu\nu}(\Omega_+) - 4R_{\mu\nu}(\Omega_+)R^{\nu\mu}(\Omega_+) + R(\Omega_+)^2) \\
\epsilon\epsilon H^2 R(\Omega_+) &= \epsilon_{\alpha\mu_0\cdots\mu_4}\epsilon^{\alpha\nu_0\cdots\nu_4}H^{\mu\mu_2}{}_{\nu_0}H_{\nu_1\nu_2}{}^{\mu_0}R^{\mu_3\mu_4}{}_{\nu_3\nu_4}(\Omega_+) \\
\epsilon\epsilon H^4 &= \epsilon_{\alpha\beta\mu_1\cdots\mu_4}\epsilon^{\alpha\beta\nu_1\cdots\nu_4}H^{\mu_1\mu_2\rho}H_{\nu_1\nu_2\rho}H^{\mu_3\mu_4\sigma}H_{\nu_3\nu_4\sigma}
\end{aligned}
$$

- Only the o-o contribution receives additional corrections

The string one-loop four-point function

- In the o-o sector, we take one vertex operator in the $(-1,-1)$ picture and three in the $(0,0)$ picture

$$\mathcal{A} \sim \theta^{(1)}_{\mu_1\nu_1}\theta^{(2)}_{\mu_2\nu_2}\theta^{(3)}_{\mu_3\nu_3}\theta^{(4)}_{\mu_4\nu_4} \times \left\langle \begin{matrix} \psi\cdot\partial X(0)\,\psi^{\mu_1} \\ \tilde{\psi}\cdot\bar{\partial} X(0)\,\psi^{\nu_1} \end{matrix} \prod_{i=2}^{4} \begin{matrix} (i\partial X^{\mu_i} + \frac{1}{2}\alpha' k_i\cdot\psi\psi^{\mu_i}) \\ (-i\bar{\partial} X^{\nu_i} + \frac{1}{2}\alpha' k_i\cdot\tilde{\psi}\tilde{\psi}^{\nu_i}) \end{matrix} \prod_{i=1}^{4} e^{ik_i\cdot X} \right\rangle$$

- The computation is simplified because we must soak up six fermion zero modes on each side of the string
  - Multiply out the vertex operators, and keep the six-fermion and eight-fermion terms on each side
  - We focus on the kinematical factors, and do not explicitly perform the modular integrals
- We find

$$e^{-1}\mathcal{L}_{\text{o-o}} \sim \epsilon_6\epsilon_6(R(\Omega_+)^2 + \tfrac{4}{3}H^2R(\Omega_+) + \tfrac{1}{9}H^4)$$

in agreement with the Heterotic/IIA result

What about the six-dimensional (1,0) theory?

- The six dimensional Lagrangian supports our conjecture that only the odd-odd term picks up additional corrections beyond the introduction of a connection with torsion
- Keeping only the NSNS sector corresponds to a (1,0) truncation
- Suggests the supersymmetrization of (1,0) R^2 actions

$$
\begin{aligned}
t_4 t_4 R^2 &\rightarrow t_4 t_4 R(\Omega_-)^2 \qquad\qquad + B \wedge \mathrm{Tr}\, R(\Omega_-) \wedge R(\Omega_-) \\
-\tfrac{1}{8}\epsilon_6\epsilon_6 R^2 &\rightarrow -\tfrac{1}{8}\epsilon_6\epsilon_6 [R(\Omega_+)^2 + \tfrac{4}{3} H^2 R(\Omega_+) + \tfrac{1}{9} H^4] \\
&\qquad\qquad\qquad + B \wedge \mathrm{Tr}\, R(\Omega_+) \wedge R(\Omega_+)
\end{aligned}
$$

- Supersymmetric Gauss-Bonnet in six dimensions?

Summary

- We have shown evidence that $R \to R(\Omega_+)$ goes a long way towards completing the higher curvature corrections in string theory
- What is missing
  - The explicit six-dimensional can be lifted to ten dimensions, but this only goes up to $H^4 R^2$
  - We have focused on the NSNS sector only
  - Partial check on T-duality in the CP-even sector, but the expressions are long and the results are incomplete
  - Supersymmetry has played only a marginal role in what we have done
  - What about the tree-level higher derivative terms?

Further thoughts

- Can we determine the full eight-derivative invariant up to H^8?
  - A direct determination would require knowledge of the eight-point scattering amplitude
- What about the RR fields?
  - We may be able to lift to eleven dimensions, and the reduce to automatically determine the RR sector interactions
- Possible applications?
  - Avoiding no-go theorems for flux compactifications by turning on H flux
  - Compactifications and lower-dimensional higher derivative couplings
  - Implications for AdS/CFT at finite N and finite λ
- How does this tie in with generalized geometry and possible hidden symmetries of string theory?