Running with Rugby Balls and Accidental SUSY

Photo: Isabelle Meltzer-Pellmann

Natural Hierarchies without Standard Model superpartners

w Alberto Salvio
Leo van Nierop
Matt Williams

The message:

- The cosmological constant problem is telling us that there must be two micron-sized dimensions (plus possibly more smaller ones)

The message: *doubling down*

- The cosmological constant problem is telling us that there must be two micron-sized dimensions (plus possibly more smaller ones)
- These dimensions must be supersymmetric (but do *NOT* require the MSSM)

“...when you have eliminated the impossible, whatever remains, however improbable, must be the truth.”

A. Conan Doyle

The message:

- The cosmological constant problem is telling us that the … zed dimensio… aller ones)
- These dim… mmetric (but need *NOT* require the MSSM)

$$\lim_{r\to 0}\left(r\frac{\partial\varphi}{\partial r}\right) = \kappa^2\left(\frac{\delta S_b}{\delta\varphi}\right)$$

- Technical progress: *back-reaction for higher codimension objects; bulk renormalization of higher codimension brane actions*

Outline

- Hierarchy problems
  - SUSY cancellations vs superpartners

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- How extra dimensions can help
  - Codimension-two backreaction
  - Why SUSY is also required

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  - Codimension-two backreaction
  - Why SUSY is also required
- Brane and bulk loops
  - Accidental SUSY

Hierarchy Problems

Hierarchy problems

- Th
- Th

The Standard Model

$$
\begin{aligned}
L = {} & \bar{E}DE + \bar{L}DL + \bar{Q}DQ + \bar{U}DU + \bar{D}DD \\
& + B_{\mu\nu}B^{\mu\nu} + W^{a}_{\mu\nu}W_{a}^{\mu\nu} + G^{\alpha}_{\mu\nu}G_{\alpha}^{\mu\nu} + G^{\alpha}_{\mu\nu}\tilde{G}_{\alpha}^{\mu\nu} \\
& + H\left(\bar{L}y_{l}E\right) + H\left(\bar{Q}y_{d}D\right) + H^{*}\left(\bar{Q}y_{u}U\right) \\
& + D_{\mu}H^{*}D^{\mu}H + \lambda\left(H^{*}H - m^{2}\right)^{2}
\end{aligned}
$$

Most general renormalizable theory possible given the particle content

Hierarchy problems

- Th

- Th

- Ideas for what lies beyond the Standard Model are largely driven by ‘technical naturalness’.
  - Motivated by belief SM is an effective field theory.

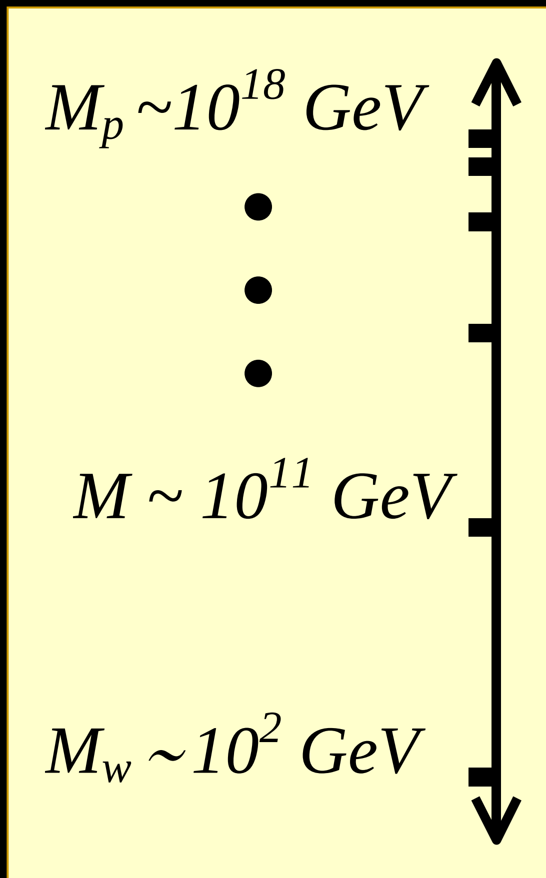
$$L_{SM} = m^2{}_0 H^* H + dimensionless$$

$$m^2 = m^2{}_0 + higher\ order \sim (126\ GeV)^2$$

Hierarchy problems

- Th … Model … ss'.

theory.



BUT: effective theory can be defined at many scales

$$m^2 \approx m_0^2 + \cdots$$

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Hierarchy problems

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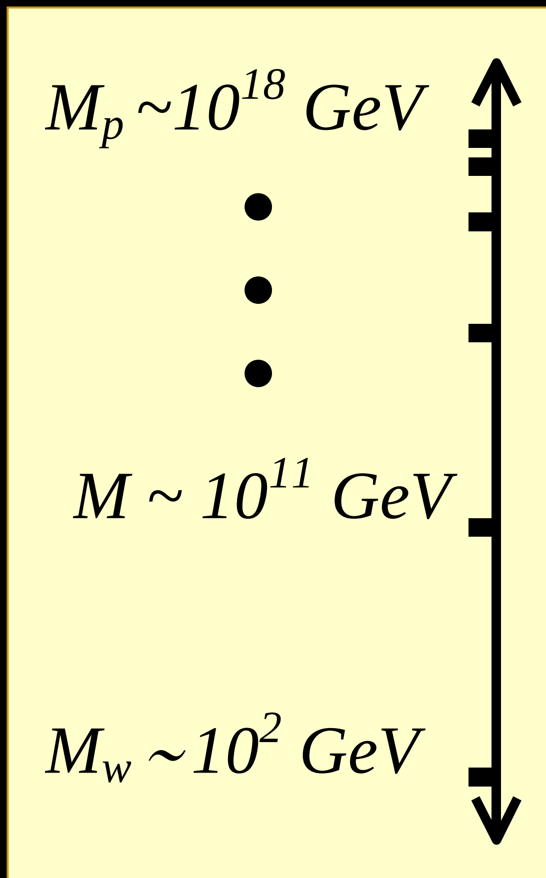


BUT: effective theory can be defined at many scales

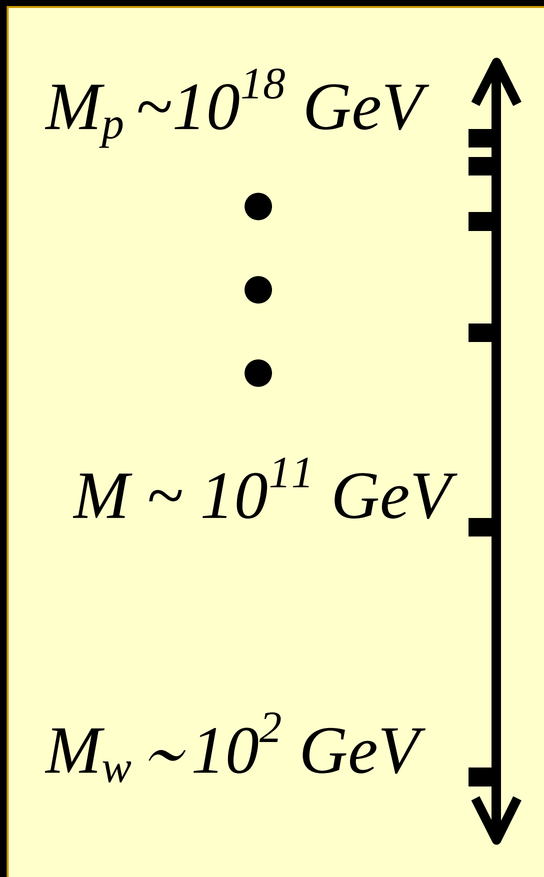
$$m^2 \approx m_1^2 + kM^2 + \cdots$$

$$m^2 \approx m_0^2 + \cdots$$

- Th

Hierarchy problems

- Th… …Model …ss'. …theory.
- Th…



BUT: effective theory can be defined at many scales

$$m^2 \approx m_1^2 + kM^2 + \cdots$$

$$m^2 \approx m_0^2$$

Must cancel to 20 decimal places!!

Hierarchy problems

- The electroweak hierarchy

- The cosmological constant

Hierarchy problems

- Th
- Th

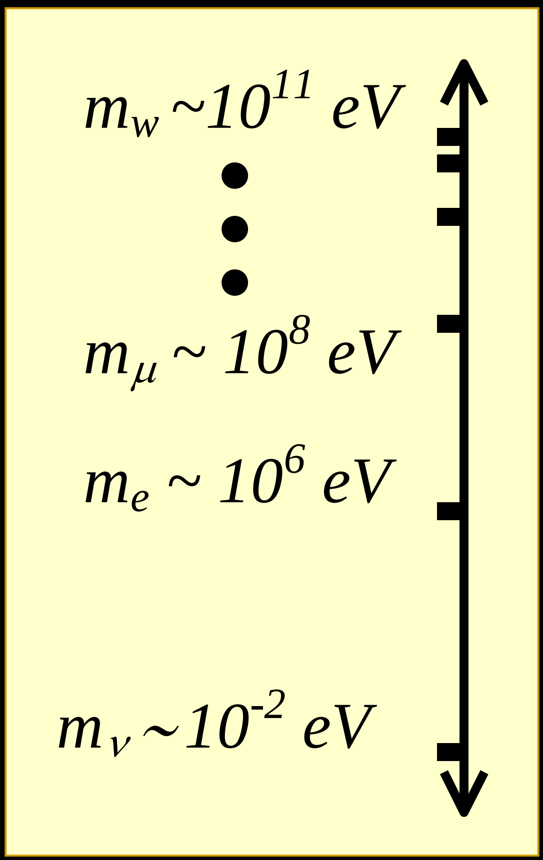
- But the SM has another unnatural parameter
  - Even more unnatural than the EW hierarchy.

$$L_{SM} = \mu^4{}_0 + m^2{}_0\, H^*H + dimensionless$$

$$\mu^4 = \mu^4{}_0 + higher\ order \sim (3 \times 10^{-3}\ eV)^4$$

Hierarchy problems

- But the SM has another unnatural parameter



Can apply same argument to scales between TeV and sub-eV scales.

$$\mu^4 \approx \mu_0^4 + k_\nu m_\nu^4$$

Hierarchy problems

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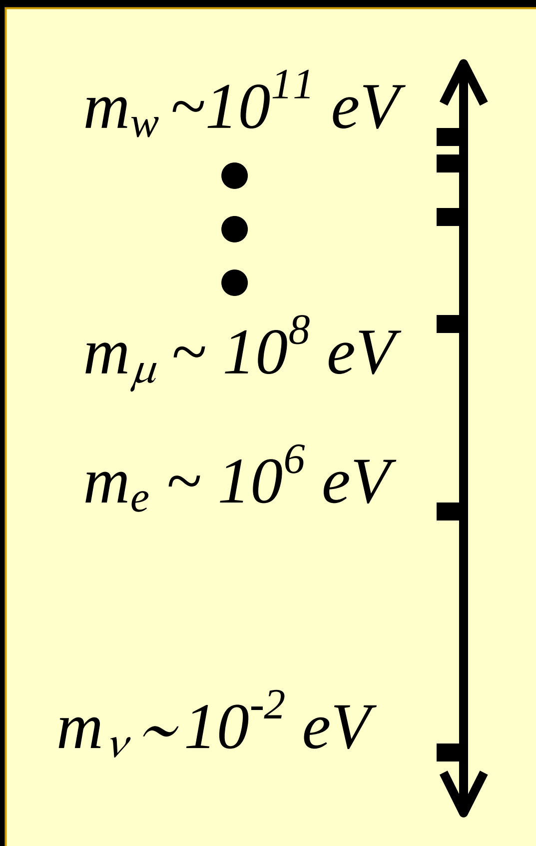

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$$\mu^4 \approx \underbrace{\mu_1^4 + k_e m_e^4}_{} + k_\nu m_\nu^4$$

$$\mu^4 \approx \mu_0^4 + k_\nu m_\nu^4$$

Hierarchy problems

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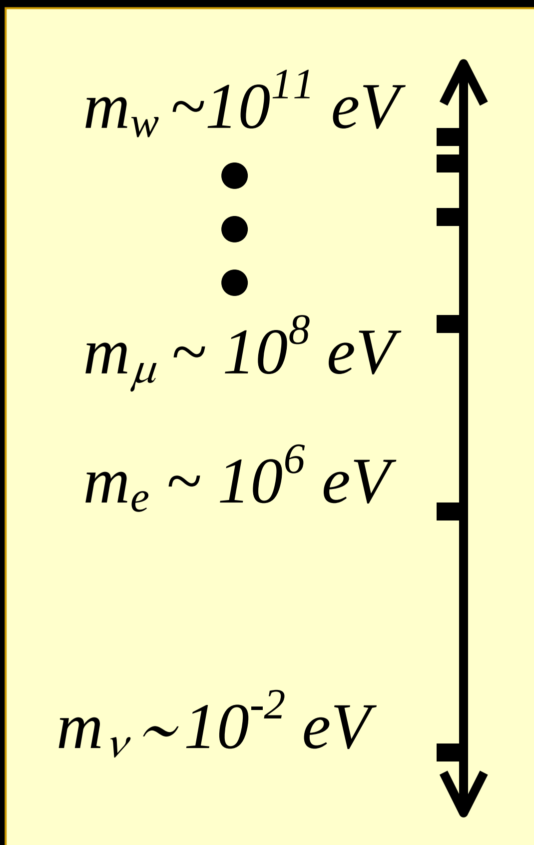


Can apply same argument to scales between TeV and sub-eV scales.

$$\mu^4 \approx \mu_1^4 + k_e m_e^4 + k_\nu m_\nu^4$$

$$\mu^4 \approx \mu_0^4 + k_\nu m_\nu^4$$

Must cancel to 32 decimal places!!

Extra Dimensions

'Towards a Naturally Small Cosmological Constant…'
Y Aghababaie, CB, S Parameswaran & F Quevedo,
hep-th:0304256

'Large Dimensions and Small Curvatures….'
CB & L van Nierop
arXiV:1101.0152

'Tecnically Natural Cosmological Constant….'
CB & L van Nierop
arXiV:1108.0345

Helpful extra dimensions

- Ge
- Ar

- The Problem:
  - Einstein's equations make a lorentz-invariant vacuum energy *(which is generically large)* an obstruction to a close-to-flat spacetime *(which we see around us)*

$$T_{\mu\nu} = \lambda \, g_{\mu\nu}$$

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

Helpful extra dimensions

Arkani-Hamed et al
Kachru et al
Carroll & Guica
Aghababaie et al

- Ge
- Ar

- The Problem:
  - Einstein's equations make a lorentz-invariant vacuum
    ene
    a cl

But this need not be true if there are more than 4 dimensions!!

$$T_{\mu\nu} = \lambda \, g_{\mu\nu}$$

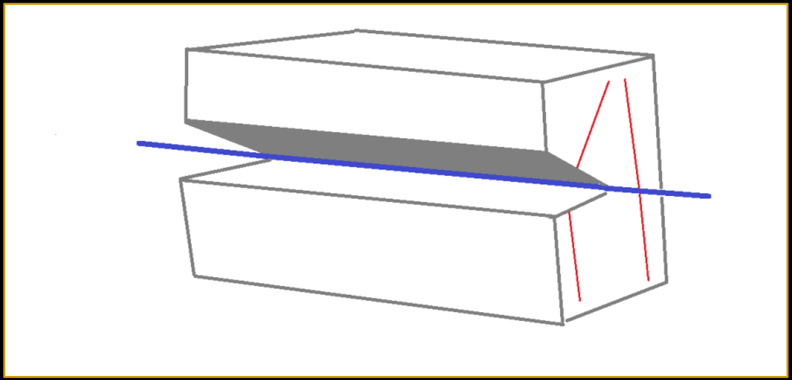
$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

Helpful extra dimensions

Vilenkin et al

- Ge
- Ar

- Why not?
  - Need not be lorentz invariant in the extra dimensions
  - Vacuum energy might curve extra dimensions, rather than the ones we *see (eg gravity field of a cosmic string)*

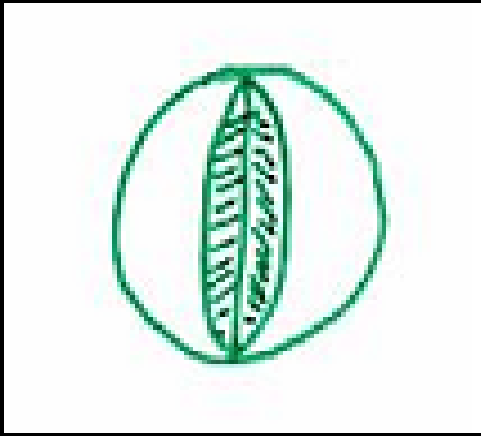


Helpful extra dimensions

Carroll & Guica
Aghababaie et al

- Ge
- Ar

- A higher-dimensional analog:
  - Similar *(classical)* examples also with a 4D brane in two extra dimensions: *e.g. the rugby ball and related solutions*



Helpful extra dimensions

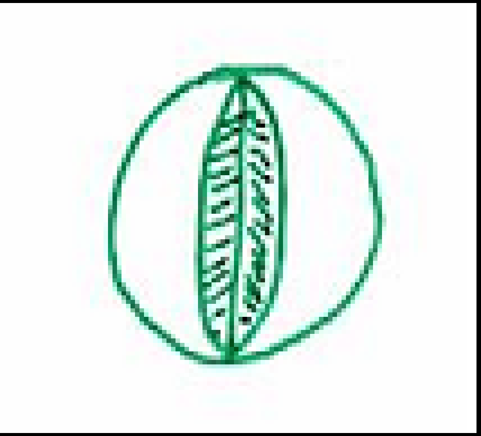
Chen, Luty & Ponton

- Ge
- An

- A higher-dimensional analog:
  - Similar *(classical)* examples also with a 4D brane in two extra dimensions: *e.g. the rugby ball and related solutions*

$$R = -2\kappa^2 \sum T_i \; \delta^2(x_i)$$

$$\text{4D cc} = \sum T_i + \frac{1}{2\kappa^2} \int d^2x \, R$$
$$= 0 \;\; \text{for all } T_i$$



A Simple Model

- Bulk: 6D Einstein-Maxwell-scalar system

$$L = \frac{1}{2\kappa^2}\left[R + (\partial\phi)^2\right] + e^{-a\phi} F_{mn}F^{mn} + V(\phi)$$

- Two specific cases
  - 6D axion: $a = 0$ and $V = \Lambda$
  - 6D supergravity: $a = 1$ and $V = \frac{2g_R^2}{\kappa^4} e^{\phi}$

A Simple Model

- Brane: Generic brane-bulk coupling

$$L_b = T(\phi) + A(\phi) * F + \cdots$$

- Interpretation:
  - *T* represents brane tension
  - *A* represents brane-localized flux

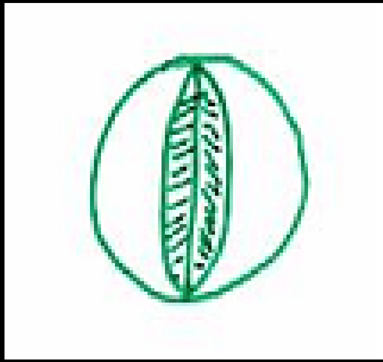
$$\frac{n}{g} = \int F + \sum_b A_b e^{\phi}$$

A Simple Model

- Simple solution

$$ds^2 = \hat{g}_{mn} dx^m \, dx^n + [dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2] e^{-a\phi_0}$$

$$F_{r\theta} = Q\alpha L \sin\left(\frac{r}{L}\right) e^{-a\phi_0} \qquad \phi = \phi_0$$



A Simple Model

Carroll & Guica
Aghababaie et al

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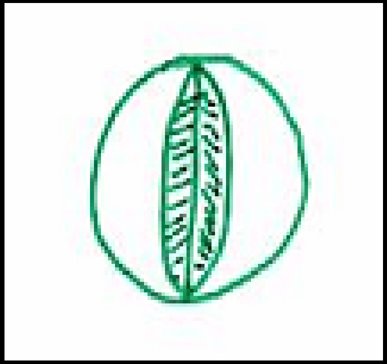

Magnetic flux required to stabilize extra dimensions against gravitational collapse

A Simple Model

Carroll & Guica
Aghababaie et al

- Simple solution

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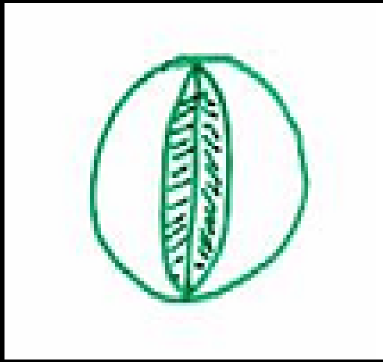

Labels flat direction *(which exists due to shift symmetry or scale invariance)*

A Simple Model

Aghababaie et al

- Simple solution

$$ds^2 = \hat{g}_{mn} dx^m\, dx^n + [dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2] e^{-a\phi_0}$$

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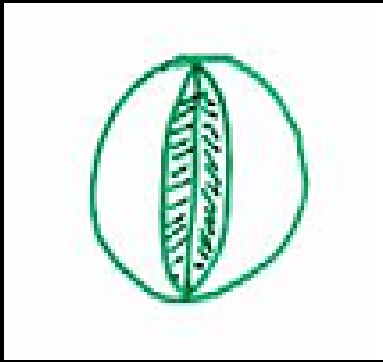

For later: notice radius is exponential in the flat direction ϕ_0 in the SUSY case

An exact classical result

- For 6D flux-stabilized supergravity we have

$$\frac{1}{2\kappa^2}\int R = S_{on-shell} = \frac{1}{2\kappa^2}\int \nabla^2\phi \propto \frac{\delta S_b}{\delta\phi}$$

and so $R = 0$ if no branes couple to 6D dilaton ϕ

- Seems to imply geometry should be robustly flat, regardless of on-brane loops and perturbations

Loops

'Running with Rugby Balls'
M Williams, CB, L van Nierop & A Salvio,
arXiV:1210.3735

'Accidental SUSY'
CB, L van Nierop, S Parameswaran, A Salvio & M Williams
arXiV:1210.5405

Brane and Bulk Loops

- UV sensitivity

- Accidental SUSY

Brane and Bulk Loops

- UV
- Ac

- Brane loops:
  - Include a massive brane-localized field and integrate it out, keeping track of dependence on *M*

$$L = T + (\partial h)^2 + M^2 h^2$$

$$\delta L = \frac{M^4}{(4\pi)^2} + \cdots$$

So no curvature if M independent of ϕ

Brane and Bulk Loops

- UV

- Bulk loops:
  - Integrate out a massive bulk field, keeping track of dependence of brane and bulk dependence on *M*

$$L_B = e^{-2\phi}[R + (\partial H)^2 + M^2 H^2]$$
$$= R + (\partial H)^2 + M^2 e^{\phi} H^2$$

$$\delta L_B = \frac{M^6 e^{3\phi}}{(4\pi)^3} + \frac{M^4 e^{2\phi}}{(4\pi)^3} R + \cdots$$

Notice loops counted by $e^{2\phi} = 1/r^4$

- Ac

Brane and Bulk Loops

- UV
- Ac

- UV sensitive renormalizations of the bulk:
  - Bulk renormalizations are insensitive to the brane boundary conditions, and so are the same as for the Salam Sezgin geometry without branes.

UV part of loops cancel
as if branes were not present
(so benefit from bulk supersymmetry)

Brane and Bulk Loops

- UV
- Ac

- Renormalization of branes by bulk loops:
  - Near-brane UV sensitivity captured by renormalization of brane lagrangian by bulk loops

$$L_B = e^{-2\phi}[R + (\partial H)^2 + M^2 H^2]$$
$$= R + (\partial H)^2 + M^2 e^{\phi} H^2$$

$$\delta L_b = c_1(\delta)\frac{M^4 e^{2\phi}}{(4\pi)^2} + c_2(\delta)\frac{M^2 e^{\phi}}{(4\pi)^2} R + \cdots$$

Notice ϕ dependence introduced by loops

Brane and Bulk Loops

- UV

- Ac

- UV sensitive renormalizations of the branes:
  - Bulk renormalizations are very small because of the flux-stabilizing relation between the dilaton and *r*.

$$e^{\phi} = \frac{k}{(Mr)^2} \quad \text{which implies}$$

$$\delta T = \frac{M^4 e^{2\phi}}{(4\pi)^2} = \frac{k}{(4\pi\, r^2)^2}$$

Only one bulk loop is dangerous

Brane and Bulk Loops

- UV
- Ac

- Cancellations occur once summed over 6D particle supermultiplets
  - eg F^2 *and R terms renormalize together in bulk action, such that they cancel in* L_B *when evaluated at a rugby ball*
  - *Renormalizations of brane action generically does not cancel in this way, due to supersymmetry breaking boundary conditions at the branes*
  - *Exception is when both branes are identical, in which case renormalizations of brane action also cancel*

Brane and Bulk Loops

- UV sensitivity
- Accidental SUSY

Brane and Bulk Loops

- UV
- Ac

- For equal brane case one bulk SUSY turns out to remain preserved even once brane back-reaction is included
  - In absence of branes Killing spinor condition can be solved because spin connection cancels gauge R symmetry connection of background flux
  - *For pure tension branes the brane boundary condition excludes the resulting Killing spinor*
  - *When $L_b = T + A * F$ then SUSY not broken if $T + A\, e^{\phi} = 0$, but this is automatic from flux quantization.*

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  - Little explored beyond codimension 1

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  - Little explored beyond codimension 1
- Vast unexplored territory
  - Codim-2 back-reaction as big as brane effects
  - Promising for naturalness issues (different parametric dependences in energy; unusual stability to quantum corrections; etc)

Conclusions

- Branes and brane back-reaction can have important implications for low-energy theory
  - Little explored beyond codimension 1
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  - Codim-2 back-react
  - Promising for natur
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Potentially wide-ranging observational implications for Dark Energy cosmology, the LHC and elsewhere…

Fin