Running with Rugby Balls and Accidental SUSY

Natural Hierarchies without Standard Model superpartners

Photo: Isabelle Meltzer-Pellmann
The message:

- The cosmological constant problem is telling us that there must be two micron-sized dimensions (plus possibly more smaller ones)
The message: *doubling down*

- The cosmological constant problem is telling us that there must be two micron-sized dimensions (plus possibly more smaller ones)
- These dimensions must be supersymmetric (but do *NOT* require the MSSM)
“...when you have eliminated the impossible, whatever remains, however improbable, must be the truth.”

A. Conan Doyle
The message:

• The cosmological constant problem is telling us that there must be two micron-sized dimensions (plus possibly more smaller ones).

• These dimensions must be supersymmetric (but need NOT require the MSSM).

• Technical progress: *back-reaction for higher codimension objects; bulk renormalization of higher codimension brane actions*
Outline

• Hierarchy problems
  • SUSY cancellations vs superpartners
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- Hierarchy problems
  - SUSY cancellations vs superpartners
- How extra dimensions can help
  - Codimension-two backreaction
  - Why SUSY is also required
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• Hierarchy problems
  • SUSY cancellations vs superpartners

• How extra dimensions can help
  • Codimension-two backreaction
  • Why SUSY is also required

• Brane and bulk loops
  • Accidental SUSY
Hierarchy Problems
Hierarchy problems

The Standard Model

\[ L = \bar{E}DE + \bar{L}DL + \bar{Q}DQ + \bar{U}DU + \bar{D}DD \]
\[ + B_{\mu\nu}B^{\mu\nu} + W_{\mu\nu}W_{\alpha}^{\mu\nu} + G_{\mu\nu}G_{\alpha}^{\mu\nu} + G_{\mu\nu}^*G_{\alpha}^{\mu\nu} \]
\[ + H(\bar{L}y_lE) + H(\bar{Q}y_uD) + H^*(\bar{Q}y_uU) \]
\[ + D_\mu H^*D^{\mu}H + \lambda(H^*H - m^2)^2 \]

*Most general renormalizable theory possible given the particle content*
Hierarchy problems

• Ideas for what lies beyond the Standard Model are largely driven by ‘technical naturalness’.
  • Motivated by belief SM is an effective field theory.

\[ L_{SM} = m_0^2 H^*H + \text{dimensionless} \]

\[ m^2 = m_0^2 + \text{higher order} \sim (126 \text{ GeV})^2 \]
Hierarchy problems

- The electroweak hierarchy
- The cosmological constant

Ideas for what lies beyond the Standard Model are largely driven by ‘technical naturalness’. Motivated by belief, the Standard Model is an effective field theory.

\[
L_{\text{SM}} = m_0^2 H^2 + \text{dimensionless terms}
\]

\[
m^2 = m_0^2 + \ldots
\]

\[M_p \sim 10^{18} \text{ GeV}\]
\[M \sim 10^{11} \text{ GeV}\]
\[M_w \sim 10^2 \text{ GeV}\]

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BUT: effective theory can be defined at many scales.
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\[ m^2 \approx m_1^2 + k M^2 + \cdots \]

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Texas A&M 2013
Hierarchy problems

- The electroweak hierarchy
  - $L_{SM} = m_0^2 H^* H + \text{dimensionless}$
  - $m^2 = m_0^2 + \text{higher order}$
  - $M \sim 10^{11} \text{ GeV}$
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- The cosmological constant

BUT: effective theory can be defined at many scales

$m^2 \approx m_1^2 + kM^2 + \cdots$

Must cancel to 20 decimal places!!
Hierarchy problems

- The electroweak hierarchy
- The cosmological constant
Hierarchy problems

- But the SM has another unnatural parameter
  - Even more unnatural than the EW hierarchy.

\[ L_{SM} = \mu^4_0 + m^2_0 H^*H + \text{dimensionless} \]

\[ \mu^4 = \mu^4_0 + \text{higher order} \sim (3 \times 10^{-3} \text{ eV})^4 \]
Hierarchy problems

- But the SM has another unnatural parameter.

\[ L_{SM} = \mu^2 + m^2 + \text{dimensionless} \]

\[ \mu^2 = \mu^2_0 + \text{higher order} \]

\[ m_e \sim 10^6 \text{ eV} \]

\[ m_\mu \sim 10^8 \text{ eV} \]

\[ m_\nu \sim 10^{-2} \text{ eV} \]

\[ m_w \sim 10^{11} \text{ eV} \]

Can apply same argument to scales between TeV and sub-eV scales.

\[ \mu^4 \approx \mu_0^4 + k_\nu m_\nu^4 \]
Hierarchy problems

- But the SM has another unnatural parameter

\[ L_{SM} = \mu^2 + m_{10}^2 + \mu^2 + \text{dimensionless} \]

- Can apply same argument to scales between TeV and sub-eV scales.

\[ \mu^4 \approx \mu_1^4 + k_e m_e^4 + k_v m_v^4 \]

\[ \mu^4 \approx \mu_0^4 + k_v m_v^4 \]

\[ m_w \sim 10^{11} \text{ eV} \]

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Can apply same argument to scales between TeV and sub-eV scales.

\[ \mu^4 \approx \mu_1^4 + k_e m_e^4 + k_\nu m_\nu^4 \]

\[ \mu^4 \approx \mu_0^4 + k_\nu m_\nu^4 \]

Must cancel to 32 decimal places!!
Extra Dimensions

‘Towards a Naturally Small Cosmological Constant…’
Y Aghababaie, CB, S Parameswaran & F Quevedo,
hep-th:0304256

‘Large Dimensions and Small Curvatures….’
CB & L van Nierop
arXiv:1101.0152

‘Tecnically Natural Cosmological Constant….’
CB & L van Nierop
arXiv:1108.0345
Helpful extra dimensions

• The Problem:
  • Einstein’s equations make a lorentz-invariant vacuum energy *(which is generically large)* an obstruction to a close-to-flat spacetime *(which we see around us)*

\[ T_{\mu\nu} = \lambda \ g_{\mu\nu} \]

\[ G_{\mu\nu} = 8\pi G \ T_{\mu\nu} \]
Helpful extra dimensions

• The Problem:
  - Einstein’s equations make a lorentz-invariant vacuum energy an obstruction to a close-to-flat spacetime (which we see around us).
  
  \[ T_{\mu\nu} = \lambda g_{\mu\nu} \]

  But this need not be true if there are more than 4 dimensions!!

• General arguments

• An explicit realization

Arkani-Hamed et al
Kachru et al
Carroll & Guica
Aghababaie et al

\[ G_{\mu\nu} = 8\pi G T_{\mu\nu} \]
Helpful extra dimensions

Why not?

- Need not be lorentz invariant in the extra dimensions
- Vacuum energy might curve extra dimensions, rather than the ones we see (e.g., gravity field of a cosmic string)

Vilenkin et al
Helpful extra dimensions

• A higher-dimensional analog:
  • Similar (classical) examples also with a 4D brane in two extra dimensions: *e.g. the rugby ball and related solutions*
Helpful extra dimensions

- A higher-dimensional analog:
  - Similar (classical) examples also with a 4D brane in two extra dimensions: *e.g. the rugby ball and related solutions*

\[
R = -2\kappa^2 \sum T_i \delta^2(x_i)
\]

\[
4D \text{ cc} = \sum T_i + \frac{1}{2\kappa^2} \int d^2x \ R
= 0 \text{ for all } T_i
\]
A Simple Model

• **Bulk:** 6D Einstein-Maxwell-scalar system

\[ L = \frac{1}{2\kappa^2} [R + (\partial \phi)^2] + e^{-\alpha \phi} F_{mn}F^{mn} + V(\phi) \]

• **Two specific cases**
  • 6D axion: \( a = 0 \) and \( V = \Lambda \)
  • 6D supergravity: \( a = 1 \) and \( V = \frac{2g_R^2}{\kappa^4} e^\phi \)
A Simple Model

• Brane: Generic brane-bulk coupling

\[ L_b = T(\phi) + A(\phi) \ast F + \cdots \]

• Interpretation:
  • \(T\) represents brane tension
  • \(A\) represents brane-localized flux

\[ \frac{n}{g} = \int F + \sum_b A_b e^\phi \]
A Simple Model

• Simple solution

\[ ds^2 = \tilde{g}_{mn} dx^m \, dx^n + [dr^2 + \alpha^2 L^2 \sin^2 \left( \frac{r}{L} \right) d\theta^2 ] e^{-\alpha \phi_0} \]

\[ F_{r\theta} = Q\alpha L \sin \left( \frac{r}{L} \right) e^{-\alpha \phi_0} \quad \phi = \phi_0 \]
A Simple Model

• Simple solution

\[ ds^2 = \hat{g}_{mn} dx^m dx^n + [dr^2 + \alpha^2 L^2 \sin^2 \left(\frac{r}{L}\right) d\theta^2] e^{-\alpha \phi_0} \]

\[ F_{r\theta} = Q\alpha L \sin \left(\frac{r}{L}\right) e^{-\alpha \phi_0} \quad \phi = \phi_0 \]

Magnetic flux required to stabilize extra dimensions against gravitational collapse

Carroll & Guica
Aghababaie et al
A Simple Model

• Simple solution

\[
\begin{align*}
    ds^2 &= \hat{g}_{mn} dx^m \, dx^n + \left[ dr^2 + \alpha^2 L^2 \sin^2 \left( \frac{r}{L} \right) d\theta^2 \right] e^{-\alpha \phi_0} \\
    F_{r\theta} &= Q\alpha L \sin \left( \frac{r}{L} \right) e^{-\alpha \phi_0} \\
    \phi &= \phi_0
\end{align*}
\]

Labels flat direction (which exists due to shift symmetry or scale invariance)
A Simple Model

• Simple solution

\[ ds^2 = \hat{g}_{mn} dx^m dx^n + [dr^2 + \alpha^2 L^2 \sin^2 \left( \frac{r}{L} \right) d\theta^2 ] e^{-\alpha \phi_0} \]

\[ F_{r\theta} = Q\alpha L \sin \left( \frac{r}{L} \right) e^{-\alpha \phi_0} \quad \phi = \phi_0 \]

For later: notice radius is exponential in the flat direction \( \phi_0 \) in the SUSY case

Aghababaie et al
An exact classical result

- For 6D flux-stabilized supergravity we have
  \[ \frac{1}{2\kappa^2} \int R = S_{\text{on-shell}} = \frac{1}{2\kappa^2} \int \nabla^2 \phi \propto \frac{\delta S_b}{\delta \phi} \]

  and so \( R = 0 \) if no branes couple to 6D dilaton \( \phi \)

- Seems to imply geometry should be robustly flat, regardless of on-brane loops and perturbations
Loops

‘Running with Rugby Balls’
M Williams, CB, L van Nierop & A Salvio,
arXiV:1210.3735

‘Accidental SUSY’
CB, L van Nierop, S Parameswaran, A Salvio & M Williams
arXiV:1210.5405
Brane and Bulk Loops

- UV sensitivity

- Accidental SUSY
Brane and Bulk Loops

- Brane loops:
  - Include a massive brane-localized field and integrate it out, keeping track of dependence on $M$

$$L = T + (\partial h)^2 + M^2 h^2$$

$$\delta L = \frac{M^4}{(4\pi)^2} + \cdots$$

So no curvature if $M$ independent of $\phi$
Brane and Bulk Loops

- Bulk loops:
  - Integrate out a massive bulk field, keeping track of dependence of brane and bulk dependence on $M$

\[
L_B = e^{-2\phi} [R + (\partial H)^2 + M^2 H^2]
= R + (\partial H)^2 + M^2 e^{\phi} H^2
\]

\[
\delta L_B = \frac{M^6 e^{3\phi}}{(4\pi)^3} + \frac{M^4 e^{2\phi}}{(4\pi)^3} R + \cdots
\]

Notice loops counted by $e^{2\phi} = 1/r^4$
Brane and Bulk Loops

- UV sensitive renormalizations of the bulk:
  - Bulk renormalizations are insensitive to the brane boundary conditions, and so are the same as for the Salam Sezgin geometry without branes.

$UV$ part of loops cancel as if branes were not present (so benefit from bulk supersymmetry)
Brane and Bulk Loops

- Renormalization of branes by bulk loops:
  - Near-brane UV sensitivity captured by renormalization of brane lagrangian by bulk loops

\[
L_B = e^{-2\phi} [R + (\partial H)^2 + M^2 H^2]
= R + (\partial H)^2 + M^2 e^\phi H^2
\]

\[
\delta L_b = c_1(\delta) \frac{M^4 e^{2\phi}}{(4\pi)^2} + c_2(\delta) \frac{M^2 e^\phi}{(4\pi)^2} R + \ldots
\]

Notice \( \phi \) dependence introduced by loops
Brane and Bulk Loops

- UV sensitive renormalizations of the branes:
  - Bulk renormalizations are very small because of the flux-stabilizing relation between the dilaton and $r$:

\[
e^\phi = \frac{k}{(Mr)^2} \quad \text{which implies}\]

\[
\delta T = \frac{M^4e^{2\phi}}{(4\pi)^2} = \frac{k}{(4\pi r^2)^2}
\]

Only one bulk loop is dangerous
Brane and Bulk Loops

- Cancellations occur once summed over 6D particle supermultiplets
  - eg $F^2$ and $R$ terms renormalize together in bulk action, such that they cancel in $L_B$ when evaluated at a rugby ball
  - Renormalizations of brane action generically does not cancel in this way, due to supersymmetry breaking boundary conditions at the branes
  - Exception is when both branes are identical, in which case renormalizations of brane action also cancel
Brane and Bulk Loops

- UV sensitivity

- Accidental SUSY
Brane and Bulk Loops

• For equal brane case one bulk SUSY turns out to remain preserved even once brane back-reaction is included
  • In absence of branes Killing spinor condition can be solved because spin connection cancels gauge R symmetry connection of background flux
  • For pure tension branes the brane boundary condition excludes the resulting Killing spinor
  • When $L_b = T + A \ast F$ then SUSY not broken if $T + A e^{\Phi} = 0$, but this is automatic from flux quantization.
Conclusions
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  • Little explored beyond codimension 1
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• Branes and brane back-reaction can have important implications for low-energy theory
  • Little explored beyond codimension 1
• Vast unexplored territory
  • Codim-2 back-reaction as big as brane effects
  • Promising for naturalness issues (different parametric dependences in energy; unusual stability to quantum corrections; etc)
Conclusions

- Branes and brane back-reaction can have important implications for low-energy theory
  - Little explored beyond codimension 1
- Vast unexplored territory
  - Codim-2 back-reaction
  - Promising for naturalness issues (different parametric dependences in energy; unusual stability to quantum corrections; etc)

Potentially wide-ranging observational implications for Dark Energy cosmology, the LHC and elsewhere...