Quantum quenches and the entropy production in strongly coupled systems

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$\Rightarrow$ Consider quantum mechanics with Hamiltonian dependent on an external parameter λ,

$$H_\lambda = H(\hat{p}, \hat{x}; \lambda)\,.$$

The dynamics of the system induced by variation in λ is well-understood:

- For a stationary state $|n\rangle$ with energy $E_n = \hbar\omega_n$, the *slow* changes in λ, *i.e.*, $\frac{d\ln\lambda(t)}{\omega_n dt} \ll 1$, are adiabatic: the system continues to be in the state $|n\rangle$ with time-dependent energy $E_n = E_n(\lambda(t))$ tracing the change in λ.
- A *fast* (abrupt) change in λ, *i.e.*, $\frac{d\ln\lambda(t)}{dt} = C\cdot\delta(t)$ results in the evolution of the wave-function ψ_n of $|n\rangle$ for $t>0$ as a mixed state of *quenched* Hamiltonian

$$H_\lambda \to H_{e^C\cdot\lambda}\,.$$

What about QFT?

The behavior of quantum quenches in QFT is a much more difficult question, *i.e.*, the dynamics of the four dimensional quantum field theory under time-dependent variation of one of its coupling constants,

$$\mathcal{L}_0 \ \to \mathcal{L}_\lambda = \mathcal{L}_0 + \lambda(t)\ \mathcal{O}\,.$$

Here, $\mathcal{L}_0$ is the undeformed Lagrangian of the theory, and $\lambda(t)$ is a time-dependent coupling constant of a relevant operator $\mathcal{O}$ in the theory. A textbook example in QFT — an interaction picture — is when $\mathcal{L}_0$ is a Lagrangian of a free theory, and the (small) coupling constant λ is turned-on adiabatically so that

$$\lim_{t\to -\infty} \lambda(t) = 0\,, \qquad \lim_{t\to +\infty} \frac{d\ln\lambda}{dt} = 0\,.$$

Description of quantum quenches in strongly interactive systems, or with non-adiabatic profile of a coupling constant, has been studied to a lesser extent.

Some questions one can be interested in:

- How transition between the adiabatic and non-adiabatic regimes occur?
- What are the observables of a non-stationary QFTs?
- Are instantaneous quenches in QFT well-defined?
- How does a system relaxes as a result of a quench?
- Is there a difference in relaxation of one-point and many-point correlation functions?
- How does non-local observables (Wilson lines) relax?
- $\cdots$

Outline of the talk:

- Thermal Quantum Quenches in non-interactive models
  - SHO
  - Bosonic free field theory
- Description of the interactive model: mass-deformed $\mathcal{N} = 4$ SYM
- Main tool: gauge/gravity correspondence
  - Holographic renormalization and ambiguities
- Results:
  - typical response of the system to a quench;
  - non-abiabaticity of the quench;
  - no instantaneous quenches;
  - renormalization scheme-dependence and divergences in $\langle T_{\mu\nu} \rangle$ and $\mathcal{O}_\Delta$;
  - renormalization scheme-dependence and the relaxation time;
  - constructing renormalization scheme-independent observables.
- Future directions

$\Longrightarrow$ Follow Sotiriadis, Calabrese & Cardy (arXiv:0903.0895)

Consider SHO:

$$H_0 = \frac{1}{2}\pi^2 + \frac{1}{2}\omega_0^2\phi^2$$

quenched abruptly at $t = 0$ as $\omega_0 \to w$. Basic assumption:

$$\langle \cdots \rangle|_{t=0_-} = \langle \cdots \rangle|_{t=0_+}$$

all physical observables are continuous across the quench.

In the thermal state $\beta_0 = \frac{1}{T_0}$,

$$\langle \phi^2 \rangle_{\beta_0} = \frac{1}{2\omega_0} \coth \frac{\beta_0 \omega_0}{2}\,, \qquad \langle \pi^2 \rangle_{\beta_0} = \frac{\omega_0}{2} \coth \frac{\beta_0 \omega_0}{2}$$

$$\langle H_0 \rangle_{\beta_0} = \frac{1}{2}\omega_0 \coth \frac{\beta_0 \omega_0}{2} \qquad \Longrightarrow \qquad \langle H \rangle_{\beta_0} = \frac{\omega^2 + \omega_0^2}{4\omega_0} \coth \frac{\beta_0 \omega_0}{2}$$

$$\frac{E - E_0}{E_0} = \frac{1}{2}\left(\left(\frac{\omega}{\omega_0}\right)^2 - 1\right)$$

$\Longrightarrow$ In abrupt changes the work done on a system be positive/negative

$\Longrightarrow$ Bosonic free field theory:

$$H_0 = \int d^3k \left(\frac{1}{2}\pi_k^2 + \frac{1}{2}\omega_{0,k}^2 \phi_k^2 \right) , \qquad \omega_{0,k} = \sqrt{m_0^2 + k^2}$$

Again, abrupt quench at $t = 0$ with $m_0 \to m$ (same assumptions as in SHO).

It is straightforward to compute mixed momentum-time correlator in a quench:

$$C_{\beta_0}(k; t_1, t_2) \equiv \langle \mathcal{T}\{\phi_k(t_1)\phi_k(t_2)\}\rangle_{\beta_0}$$

$$= \frac{1}{2\omega_k} e^{-i\omega_k|t_1-t_2|} + \left[\frac{\omega_{0,k}}{4} \left(\frac{1}{\omega_{0,k}^2} + \frac{1}{\omega_k^2} \right) \coth \frac{\beta_0 \omega_{0,k}}{2} - \frac{1}{2\omega_k} \right] \cos \omega_k (t_1 - t_2)$$

$$+ \frac{\omega_{0,k}}{4} \left(\frac{1}{\omega_{0,k}^2} - \frac{1}{\omega_k^2} \right) \coth \frac{\beta_0 \omega_{0,k}}{2} \cos \omega_k (t_1 + t_2)$$

$\Longrightarrow$ It can be argued that at late times $\{t_1, t_2\} \to \infty$ the last (time-translationary non-invariant) term can be neglected.

Thus, the quenched propagator at late times takes form

$$C_{\beta_0}(k;t_1,t_2)\Big|^{quenched} = \frac{1}{2\omega_k}e^{-i\omega_k|t_1-t_2|} + \mathcal{A}(\beta_0,\omega_{0,k},\omega_k)\ \cos\omega_k(t_1-t_2)$$

$$C_{\beta}(k;t_1,t_2)\Big|^{thermal} = \frac{1}{2\omega_k}e^{-i\omega_k|t_1-t_2|} + \frac{1}{\omega_k(e^{\beta\omega_k}-1)}\ \cos\omega_k(t_1-t_2)$$

Comparing the second terms we obtain the *effective temperature* $\beta_{eff}(k)$ after the quench:

$$\beta_{eff}(k) = \frac{1}{\omega_k}\ \ln\left(1+\frac{1}{\omega_k\mathcal{A}}\right)$$

In the "hot quench" limit, $\frac{1}{\beta_0} \gg \omega_{0,k}$

$$\beta_{eff}(k) = \frac{2\beta_0}{1+\omega_k^2/\omega_{0,k}^2}$$

Further in the limit $m_0 \gg m$

$$\beta_{eff} = 2\beta_0$$

i.e., , the final temperature is one-half the initial.

$\Longrightarrow$Along these lines, one can treat abrupt quenches in weakly interactive models

Rest of the talk:

- quenches in interactive theory (true thermalization);
- we consider quenches *smoothed* over a time-scale $\tau \propto \alpha\beta_0$ (for some constant α), and take a limit of "abrupt quench" as $\alpha \to 0$

Basic AdS/CFT correspondence:

gauge theory		string theory
$\mathcal{N}=4$ $SU(N)$ SYM	$\Longleftrightarrow$	N-units of 5-form flux in type IIB string theory
g^2_{YM}	$\Longleftrightarrow$	g_s

$\Longrightarrow$ Each of the duality frames are valid in complimentary regimes. In the 't Hooft limit (planar limit), $N \to \infty$, $g^2_{YM} \to 0$ with $N g^2_{YM}$ kept fixed:

- for $g^2_{YM} N \ll 1$ we can use a standard perturbation theory
- for $g^2_{YM} N \gg 1$ we can use effective supergravity description of type IIB string theory on $AdS_5 \times S^5$

$\Longrightarrow$ In the above regime we can incorporate corrections:

$$\frac{1}{N}\text{-corrections} \quad \Longleftrightarrow \quad g_s\text{-corrections}$$

$$\frac{1}{N g^2_{YM}}\text{-corrections} \quad \Longleftrightarrow \quad \alpha'\text{-corrections}$$

$\Longrightarrow$ The 'basic' holographic correspondence was extended to:

- non-conformal examples of gauge/string correspondence
- gauge theories in various dimensions
- beyond correspondence in vacuum — thermal states, near-equilibrium, etc

There are 2-ways to discuss gauge/gravity correspondence

- (a) Assume it is valid, and extract predictions
- (b) Do strong coupling computations and test the correspondence

<u>I will go with (a)</u>

Consider quenching the coupling λ_Δ in the deformation of large-N $SU(N)$ $\mathcal{N}=4$ supersymmetric Yang-Mills by a (gauge invariant) relevant operator $\mathcal{O}_\Delta$

$$\mathcal{L}_{SYM} \qquad \to \qquad \mathcal{L}_{SYM} + \lambda_\Delta \mathcal{O}_\Delta \,.$$

- We focus on two cases when $\Delta = 2, 3$
- The initial state is a thermal state of the gauge theory plasma.
- We discussed *perturbative* quenches, *i.e.*, during the quench the coupling constant λ_Δ is always small compare to the temperature of the initial state T_i:

$$\frac{|\lambda_\Delta|}{T_i^{4-\Delta}} \ll 1\,.$$

- We allow for *non-perturbative* rates of change of $\lambda_\Delta = \lambda_\Delta(t)$:

$$\lambda_\Delta(t) = \lambda_\Delta^0 \left(\frac{1}{2} \pm \frac{1}{2} \tanh \frac{t}{\mathcal{T}}\right), \qquad \mathcal{T} = \frac{\alpha}{T_i}\,,$$

i.e., , we do not restrict values of α.

- We are interested in the basic gauge invariant observables of the theory undergoing the quantum quench: the stress-energy tensor T_{ij} and the VEV

The gravitational dual to the above quench:

$$S_5 = \frac{1}{16\pi G_5} \int d^5\xi \sqrt{-g} \left(R + 12 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \mathcal{O}(\phi^4) \right) ,$$

with

$$m^2 = \begin{cases} -3 \,, & \Longleftrightarrow \text{ corresponding operator } \mathcal{O}_3 \,, \\ -4 \,, & \Longleftrightarrow \text{ corresponding operator } \mathcal{O}_2 \,. \end{cases}$$

Since our quenches are homogeneous and isotropic in the boundary spatial directions, we assume that both the background metric and the scalar field depend only on a radial coordinate r and a time v. With the background ansatz

$$ds_5^2 = -A(v,r)\ dv^2 + \Sigma(v,r)^2\ (d\vec{x})^2 + 2drdv \,, \qquad \phi = \phi(v,r) \,,$$

From the effective gravitational action we obtain the following:

- evolution equations:

$$0 = \Sigma(\dot{\Sigma})' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2 + \frac{1}{12}m^2\phi^2\Sigma^2$$

$$0 = A'' - \frac{12}{\Sigma^2}\Sigma'\dot{\Sigma} + 4 + \phi'\dot{\phi} - \frac{1}{6}m^2\phi^2$$

$$0 = \frac{2}{A}\,(\dot{\phi})' + \frac{3\Sigma'}{\Sigma A}\,\dot{\phi} + \frac{3\phi'}{\Sigma A}\,\dot{\Sigma} - \frac{m^2}{A}\,\phi$$

- the constraint equations:

$$0 = \ddot{\Sigma} - \frac{1}{2}A'\dot{\Sigma} + \frac{1}{6}\Sigma(\dot{\phi})^2$$

$$0 = \Sigma'' + \frac{1}{6}\Sigma(\phi')^2$$

In above, for any function $h(r, v)$,

$$h' \equiv \partial_r h\,, \qquad \dot{h} \equiv \partial_v h + \frac{1}{2}A\partial_r h\,.$$

When $m^2 = -3$,

$$\phi = \frac{1}{r}\ p_0 + \frac{1}{r^2}\ \ (p_0') + \frac{1}{r^3}\left(p_2 - \left(\frac{1}{2}p_0'' + \frac{1}{6}p_0^3\right)\ln r\right) + \mathcal{O}(r^{-4}\ln r)$$

$$\Sigma = r + \mathcal{O}(r^{-1})$$

$$A = r^2 - \frac{1}{6}p_0^2 + \frac{1}{r^2}\left(a_4 + \left(\frac{1}{6}p_0 p_0'' + \frac{1}{36}p_0^4 - \frac{1}{6}(p_0')^2\right)\ln r\right) + \mathcal{O}(r^{-3}\ln r)$$

where $\{p_0, p_2, a_4\}$ are functions of v.

In addition, a constraint equation implies:

$$0 = -2a_4' + \frac{5}{27}p_0^3 p_0' + \frac{2}{3}p_0' p_2 - \frac{2}{3}p_0 p_2' - \frac{1}{9}p_0' p_0'' + \frac{4}{9}p_0 p_0'''$$

Physical meaning of $\{p_0, p_2, a_4\}$:

- a 'source' [non-normalizable component],

$$p_0 \;\propto\; \lambda_3$$

- a 'response' [normalizable component]

$$p_2 \;\sim \mathcal{O}_3$$

- Note that in the absence of the source/response the constraint implies

$$a_4' = 0 \qquad \Rightarrow \qquad \text{energy density} = \textit{constant}$$

In general, the constraint equation can be integrated to quantify the change of $\mathcal{E}$ during the quench:

$$a_4 = \mathcal{C} + \frac{5}{216} p_0(v)^4 - \frac{5}{36} (p_0(v)')^2 + \frac{2}{9} p_0(v) p_0(v)'' - \frac{1}{3} p_0(v) p_2(v) + \frac{2}{3} \int_{-\infty}^{v} ds\, p_0(s)'$$

where $\mathcal{C}$ is a constant, related to the energy density in the infinite past.

Comment on numerical procedure (all to quadratic order in the source inclusive):

- Numerically solve the PDE for the scalar $\phi(v, r)$ for a given profile of the non-normalizable component

$$p_0 = p_0(v)$$

- Numerical solution determines normalizable component

$$p_2 = p_2(v)$$

- Given $\{p_0, p_2\}$ we can integrate the constraint equation to obtain

$$a_4 = a_4(v)$$

- Once $\{p_0, p_2, a_4\}$ are determined, we translate them in QFT observables:

$$\mathcal{E} = \mathcal{E}(v)\,, \qquad \mathcal{P} = \mathcal{P}(v)\,, \qquad \mathcal{O}_3 = \mathcal{O}_3(v)$$

To compute correlation functions of gauge-invariant observables, the theory has to be regularized and renormalized:

$$S_{ct} = S_{ct}^{divergent} + S_{ct}^{finite}$$

$$S_{ct}^{divergent} = \frac{1}{16\pi G_5} \int_{\partial\mathcal{M}_5,\frac{1}{r}=\epsilon} d^4x \, \sqrt{-\gamma}\Big(6 + \frac{1}{2}\phi^2 + \frac{1}{12}\phi^4 \ln\epsilon + \frac{1}{2}\,\gamma^{ij}\partial_i\phi\partial_j\phi \, \ln\epsilon + \frac{1}{12}R^\gamma\phi^2 \, \ln\epsilon\Big)$$

$$S_{ct}^{finite} = \frac{1}{16\pi G_5} \int_{\partial\mathcal{M}_5,\frac{1}{r}=\epsilon} d^4x \, \sqrt{-\gamma}\Big(\delta_1 \, \phi^4 + \delta_2 \, \gamma^{ij}\partial_i\phi\partial_j\phi + \delta_3 \, R^\gamma\phi^2\Big)$$

where we have separated the counterterms which diverges in the limit $\epsilon = \frac{1}{r} \to 0$ from the finite counterterms.

The finite counterterms are parametrized by:

$$\delta_1 \,, \qquad \delta_2 \,, \qquad \delta_3$$

Once the theory is renormalized, we can compute 1-point correlation functions:

$$8\pi G_5\ \mathcal{E} = -\frac{3}{2}a_4 - \frac{1}{12}(p_0')^2 - \frac{1}{2}p_0 p_2 + \frac{1}{3}p_0 p_0'' + \frac{7}{288}p_0^4 + \mathcal{E}^{ambiguity}$$

$$8\pi G_5 \mathcal{P} = -\frac{1}{2}a_4 - \frac{1}{36}(p_0')^2 + \frac{1}{6}p_0 p_2 - \frac{1}{18}p_0 p_0'' + \frac{7}{864}p_0^4 + \mathcal{P}^{ambiguity}$$

$$16\pi G_5\ \langle \mathcal{O}_3 \rangle = \frac{1}{2}p_0'' - \frac{1}{12}p_0^3 - 2p_2 + \mathcal{O}_3^{ambiguity}$$

where we employ the label ^{ambiguity} to denote renormalization scheme ambiguities:

$$\mathcal{E}^{ambiguity} = \frac{1}{2}\delta_1 p_0^4 + \frac{1}{2}\delta_2 (p_0')^2\,,$$

$$\mathcal{P}^{ambiguity} = -2\delta_3 (p_0')^2 - 2\delta_3 p_0 (p_0'') - \frac{1}{2}\delta_1 p_0^4 + \frac{1}{2}\delta_2 (p_0')^2$$

$$\mathcal{O}_3^{ambiguity} = 4\delta_1 p_0^3 + 2\delta_2 p_0''\,.$$

Note that for arbitrary δ_i, the following (diffeomorphism) Ward identity,

$$\partial_i \langle T_{ij} \rangle = -\langle \mathcal{O}_3 \rangle \, \partial_j p_0 \,,$$

is equivalent to the constraint

$$0 = -2a_4' + \frac{5}{27} p_0^3 p_0' + \frac{2}{3} p_0' p_2 - \frac{2}{3} p_0 p_2' - \frac{1}{9} p_0' p_0'' + \frac{4}{9} p_0 p_0'''$$

$\Rightarrow$ We focus on the quenches of the type

$$\lim_{\tau \to \pm\infty} p_0(\tau) = constant$$

so, provided that the same is true for $p_2(\tau)$, *i.e.*,

$$\lim_{\tau \to \pm\infty} p_2(\tau) = constant$$

(numerically we verified that this is indeed the case), we have a thermal equilibrium state in the infinite past, and a thermal equilibrium state in the infinite future.

For example, if

$$\lim_{\tau\to-\infty} p_0 = 0\,, \qquad \lim_{\tau\to+\infty} p_0 = 1$$

i.e., we quench from a thermal state of a **CFT** to a thermal state of a **massive** gauge theory,

$$\mathcal{E} = \frac{3}{8}\pi^2 N^2 T_i^4 \left(1-\left(2a_4+\frac{1}{3}(p_0')^2\ \ln\frac{\pi T_i}{\Lambda_2}+\frac{1}{9}(p_0')^2+\frac{2}{3}p_0p_2\right)\frac{(m_f^0)^2}{\pi^2 T_i^2}+\mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)$$

$$\mathcal{P} = \frac{1}{8}\pi^2 N^2 T_i^4 \left(1-\left(2a_4+\frac{1}{9}(p_0')^2-\frac{2}{3}p_0p_2+\frac{2}{9}p_0p_0''-\frac{2}{3}\left(p_0p_0''+(p_0')^2\right)\ \ln\frac{\pi T_i}{\Lambda_3}\right.\right.$$
$$\left.\left.+(p_0')^2\ \ln\frac{\pi T_i}{\Lambda_2}\right)\frac{(m_f^0)^2}{\pi^2 T_i^2}+\mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)$$

$$\mathcal{O}_3 = -\frac{\sqrt{2}}{2}N^2 T_i^2 m_f^0\left(p_2-\frac{1}{4}p_0''+\frac{1}{2}\ln\frac{\pi T_i}{\Lambda_2}\ p_0''+\mathcal{O}\left(\frac{(m_f^0)^2}{T_i^2}\right)\right)$$

where

$$\delta_2 = \frac{1}{2}\ln\Lambda_2\,, \qquad \delta_3 = \frac{1}{12}\ln\ \Lambda_3$$

Note that the number of ambiguities in renormalization scheme is precisely what is needed to make sense of $\ln(T)$ terms once the gravity data is

Similarly, we can analyze the quenches

$$\lim_{\tau\to-\infty} p_0 = 1\,, \qquad \lim_{\tau\to+\infty} p_0 = 0$$

i.e., we quench from a thermal state of a **massive** gauge theory to a thermal state of a **CFT**.

Another interesting observables are:

$$\frac{T_f}{T_i} = \left(1 + \left(\pm\frac{\Gamma\left(\frac{3}{4}\right)^4}{3\pi^2} - \frac{1}{2}a_4^\infty\right)\frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)$$

$$\frac{\mathcal{E}_f}{\mathcal{E}_i} = \left(1 + \left(\pm\frac{2\Gamma\left(\frac{3}{4}\right)^4}{3\pi^2} - 2a_4^\infty\right)\frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)$$

$$\frac{\mathcal{P}_f}{\mathcal{P}_i} = \left(1 - \left(\pm\frac{2\Gamma\left(\frac{3}{4}\right)^4}{3\pi^2} + 2a_4^\infty\right)\frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)\,.$$

where

$$a_4^\infty = \lim_{\tau\to+\infty} a_4(\tau)$$

Consider quenches of the type

$$p_0 = \frac{1}{2} + \frac{1}{2} \tanh \frac{\pi T_i \ \tau}{\alpha}$$

where T_i is the initial temperature.

$\Rightarrow$ For $\alpha \gg 1$ the quenches are slow compare to a characteristic thermal scale $\propto \frac{1}{T_i}$, we expect an "adiabatic" response

$$p_2(\tau)\Big|_{adiabatic} = -\frac{\Gamma\left(\frac{3}{4}\right)^4}{\pi^2}\ p_0(\tau)$$

$\Rightarrow$ note that for the adiabatic response, from

$$0 = -2a_4' + \frac{2}{3}p_0'p_2 - \frac{2}{3}p_0p_2' - \frac{1}{9}p_0'p_0'' + \frac{4}{9}p_0p_0''' \qquad \Longrightarrow \qquad a_4' \approx 0 + \mathcal{O}(\alpha^{-2})$$

$\Rightarrow$

$$\frac{T_f}{T_i} = \left(1 + \left(\pm\frac{\Gamma\left(\frac{3}{4}\right)^4}{3\pi^2} + \mathcal{O}(\alpha^{-1})\right)\frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)$$

and similarly for $\mathcal{E}\,,\mathcal{P}$

Typical response of the system:

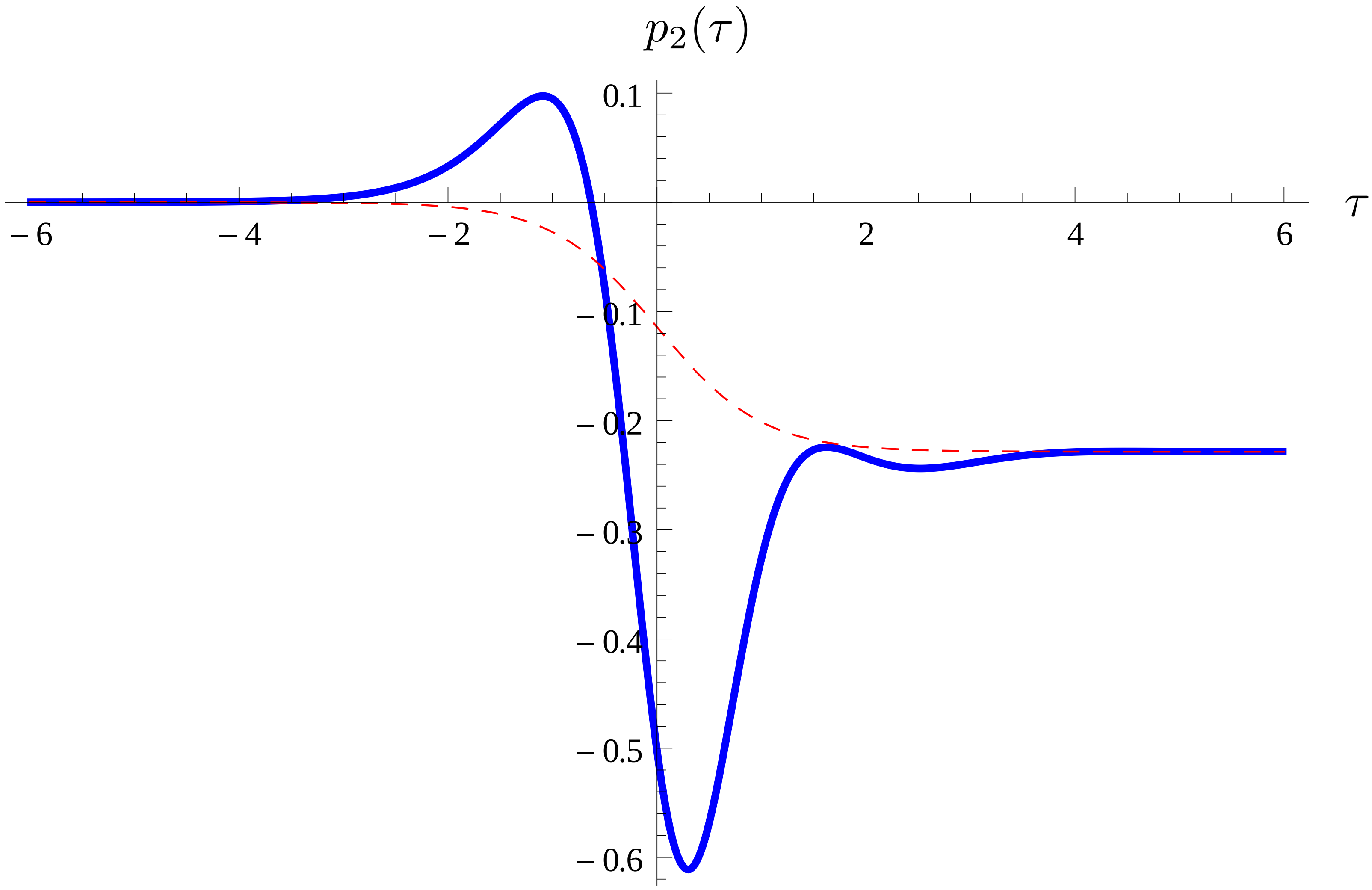
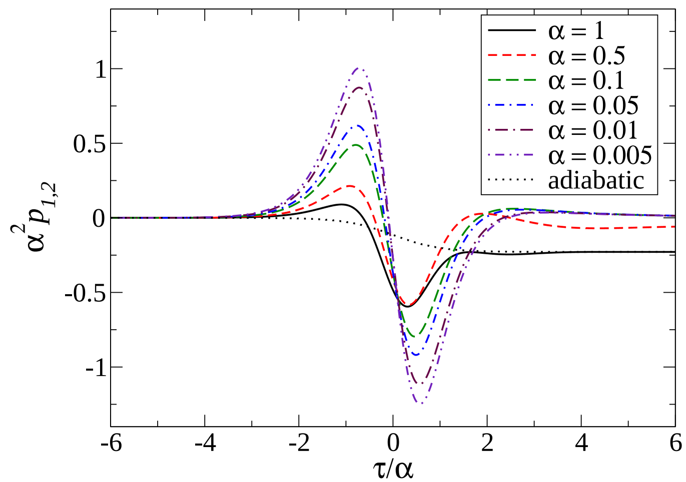
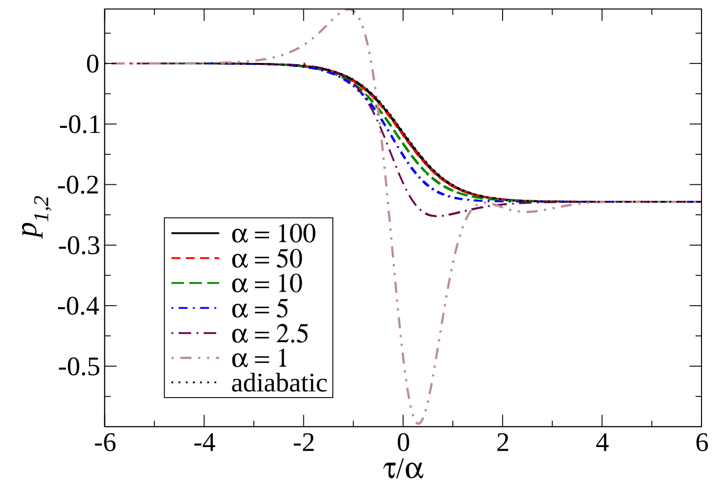
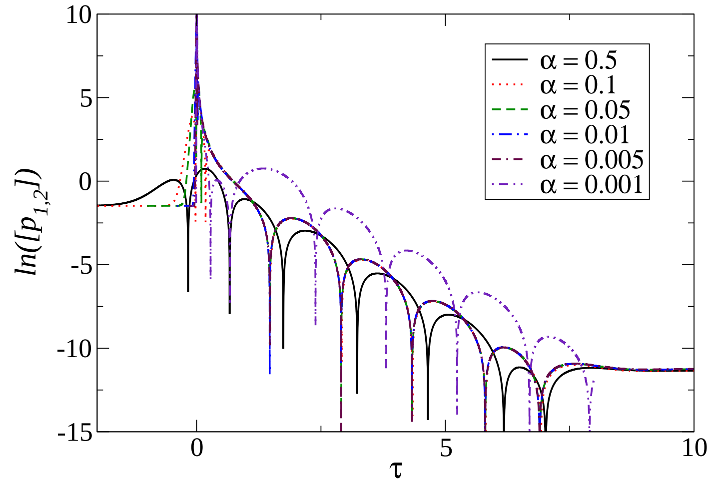


Figure 1: Evolution of the normalizable component p_2 during the quench with $\alpha = 1$. The dashed red lines represent the adiabatic response.

More evolutions:

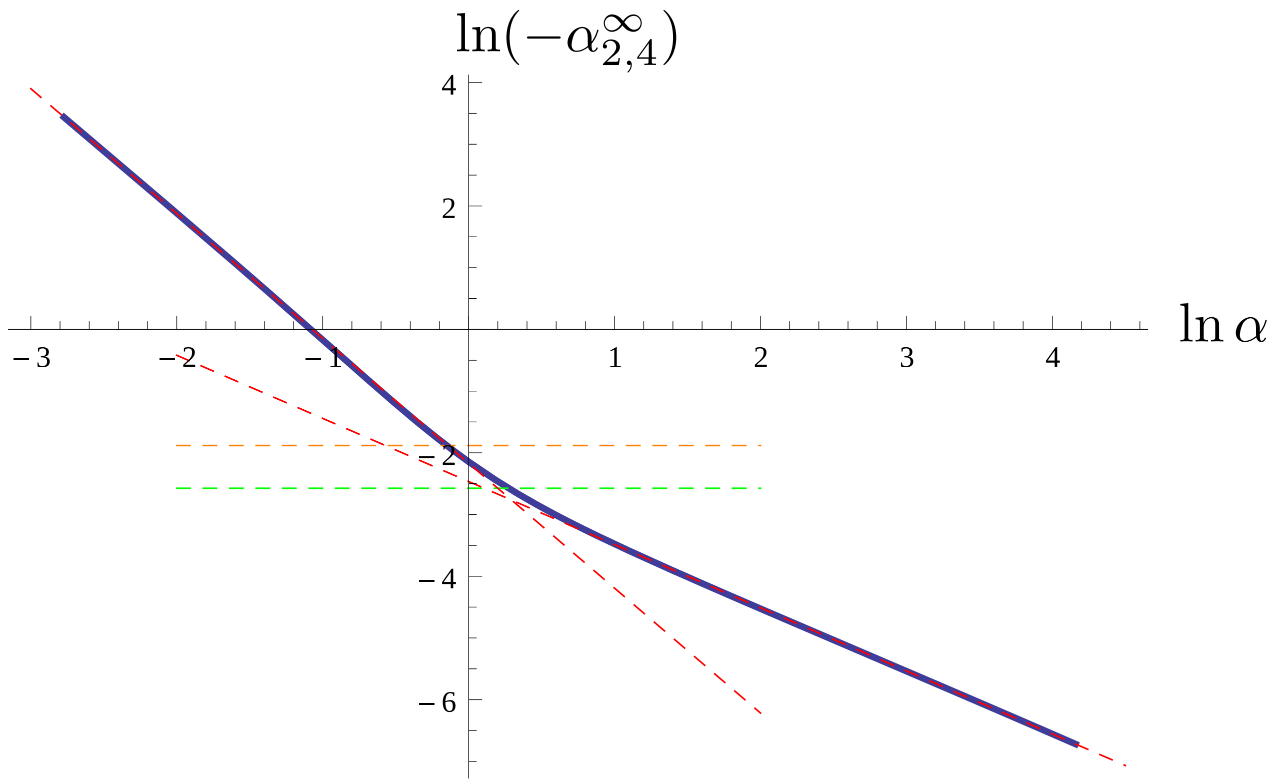






Recall:

$$\frac{T_f}{T_i} = \left(1 + \left(\pm\frac{\Gamma\left(\frac{3}{4}\right)^4}{3\pi^2} - \frac{1}{2}a_4^\infty\right)\frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)$$



$\Rightarrow$ Note that quenches *always* results in pumping energy into the system

$$\text{slow}: \qquad \ln(-a_{2,4}^{\infty})\Big|_{red,dashed}^{fit} = -2.46(5) - 1.0(2)\ \ln\alpha\,, \qquad \alpha \gg 1$$

$$\text{fast}: \qquad \ln(-a_{2,4}^{\infty})\Big|_{red,dashed}^{fit} = -2.17(0) - 2.0(2)\ \ln\alpha\,, \qquad \alpha \ll 1$$

Above asymptotic behaviour translates into

$$\frac{|\Delta T|}{T_i} \equiv \frac{|T_f - T_i|}{T_i} = \begin{cases} \propto \frac{1}{\alpha}\ \frac{(m_f^0)^2}{T_i^2},\ \alpha \gg 1 \\ \propto \frac{1}{\alpha^2}\ \frac{(m_f^0)^2}{T_i^2},\ \alpha \ll 1 \end{cases}$$

and similarly for the relative change in the energy density $\mathcal{E}$ and the pressure $\mathcal{P}$.

⇒ Note that infinitely sharp quenches

$$\alpha \to 0$$

are not allowed

Entropy production and irreversibility of quenches

$\Longrightarrow$ Forward/backwards quenches can be specified as

$$p_0^{f/b} = \frac{1}{2} \pm \frac{1}{2} \tanh \frac{\pi T_i \tau}{\alpha}$$

Since

$$p_0^f(\tau) + p_0^b(\tau) = 1 \qquad \Longrightarrow \qquad p_2^f(\tau) + p_2^b(\tau) = p_2^{equilibrium}\Big|_{p_0=1}$$

$\Longrightarrow$ Using

$$0 = -2a_4' + \frac{2}{3}p_0'p_2 - \frac{2}{3}p_0p_2' - \frac{1}{9}p_0'p_0'' + \frac{4}{9}p_0p_0'''$$

above it enough to establish

$$a_4^f(\tau) = a_4^b(\tau) \qquad \Longrightarrow \qquad a_4^{f,\infty} = a_4^{b,\infty}$$

$\Longrightarrow$ During the evolution the entropy is not well-defined. However, as system is at equilibrium at $\tau \pm \infty$ we can unambiguously compute S_i and S_f.

We find:

$$\frac{S_f}{S_i} = 1 - \frac{3}{2} a_4^{\infty} \; \frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4} \right)$$

- Since

$$a_4^{f,\infty} = a_4^{b,\infty}$$

the equilibration process is **non-reversible**;

- In all simulations

$$a_4^{\infty} < 0$$

Thus, quenches/equilibration are associated with the **entropy production**.

- Finally, recall

$$a_4^{\infty} \to 0 \qquad \text{as} \qquad \alpha \to \infty$$

Thus, there is **no entropy production** in adiabatic quenches

Quenching the coupling λ_Δ of $\mathcal{O}_\Delta$ depends on Δ!

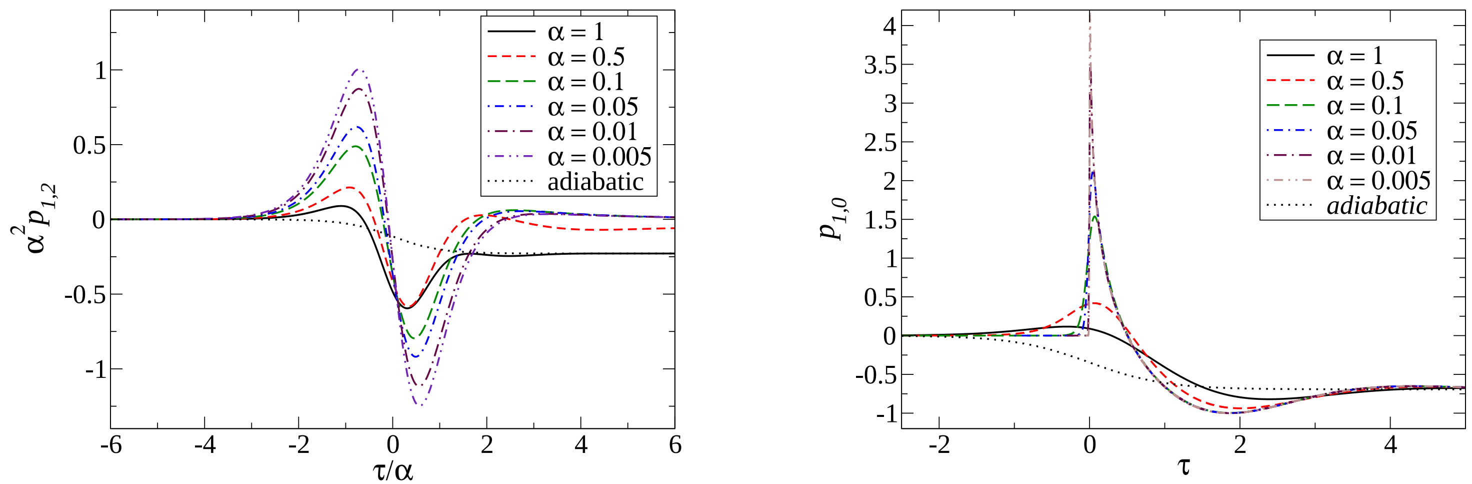


Figure 3: Left plot: $\Delta = 3$, right plot $\Delta = 2$

$$\frac{|T_f - T_i|}{T_i} = \begin{cases} \propto \frac{1}{\alpha^2} \, \frac{(m_f^0)^2}{T_i^2}, \; \alpha \ll 1 \,, & \text{when} \quad \Delta = 3 \\ \propto (-\ln \alpha) \, \frac{(m_b^0)^4}{T_i^4}, \; \alpha \ll 1 \,, & \text{when} \quad \Delta = 2 \end{cases}$$

Comment on scheme-independent observables:

- while the following observables are scheme-dependent,

$$\mathcal{E} = \frac{3}{8}\pi^2 N^2 T_i^4 \left(1 - \left(2a_4 + \frac{1}{3}(p_0')^2\ \ln\frac{\pi T_i}{\Lambda_2} + \frac{1}{9}(p_0')^2 + \frac{2}{3}p_0 p_2\right)\frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)$$

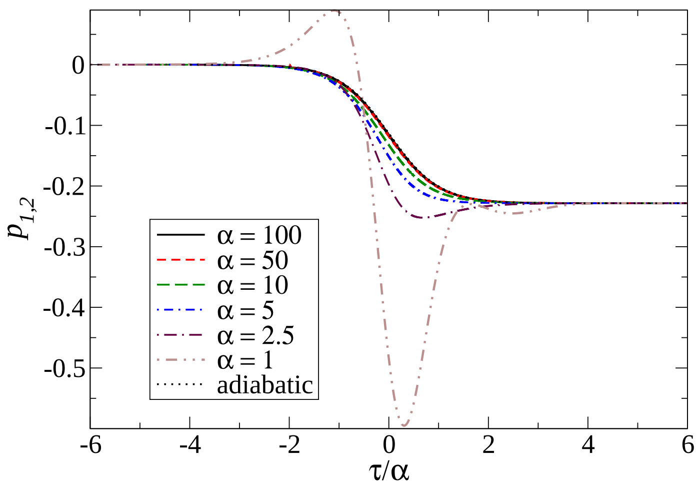
$$\mathcal{O}_3 = -\frac{\sqrt{2}}{2} N^2 T_i^2 m_f^0 \left(p_2 - \frac{1}{4}p_0'' + \frac{1}{2}\ln\frac{\pi T_i}{\Lambda_2}\ p_0'' + \mathcal{O}\left(\frac{(m_f^0)^2}{T_i^2}\right)\right)$$

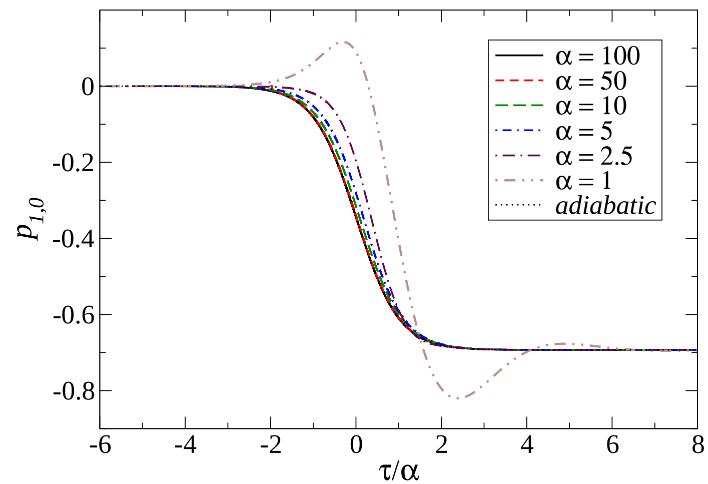
- the following combination is renormalization-scheme independent:

$$\left(\mathcal{E}(\tau) - \frac{m_f^0}{\sqrt{2}}\int_{-\infty}^{\tau} ds\ p_0'(s)\mathcal{O}_3(s)\right)$$

$\Longrightarrow$ Ideally, one would like to extract the relaxation time from renormalization-scheme independent observables.

$\Longrightarrow$ Universality of slow quenches:

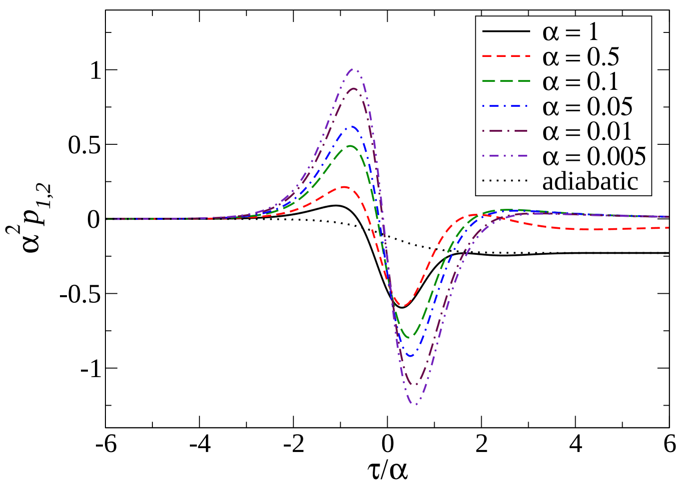


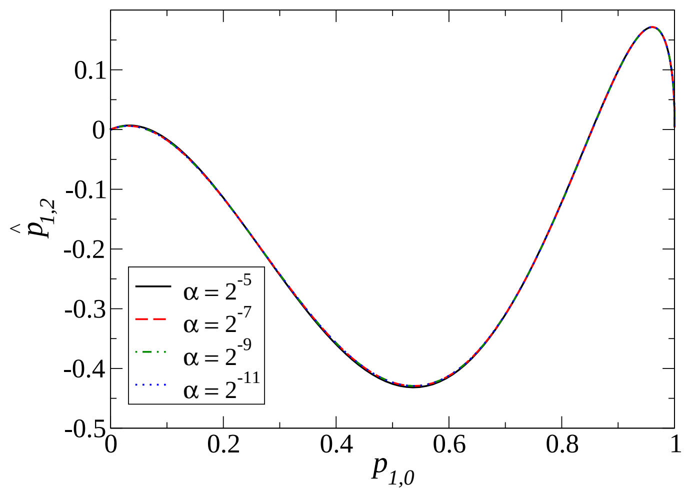


$$p_{1,2}(\tau)\Big|_{adiabatic} = -\frac{\Gamma\left(\frac{3}{4}\right)^4}{\pi^2}\; p_{1,0}(\tau)$$

$$p_{1,0}(\tau)\Big|_{adiabatic} = -\ln 2\; p^{l}_{1,0}(\tau)$$

$\Longrightarrow$ Universality of fast quenches for $\Delta = 3$:





Introduce

$$\|p_{1,2} - \mathcal{F}(\alpha)\ p''_{1,0}\| \equiv \int_0^1 d(p_{1,0})\ \left(p_{1,2} - \mathcal{F}(\alpha)\ p''_{1,0}\right)^2$$

$$= \int_{-\infty}^{\infty} d\tau\ p'_{1,0}\ \left(p_{1,2} - \mathcal{F}(\alpha)\ p''_{1,0}\right)^2$$

From the fit,

$$\mathcal{F}\Big|_{fit} \simeq 0.43 - 0.50 \;\ln \alpha$$

define a 'universal' profile

$$\hat{p}_{1,2} \equiv \alpha^2 \;\left(p_{1,2} - \mathcal{F}(\alpha)\; p''_{1,0}\right) \;,$$

Open questions:

- Fully-nonlinear quenches, not necessarily of thermal states
- Sound waves in quenches
- Quenches in various dimensions
- Non-local observables during quenches
- Quenches of SUSY couplings
- Quenches across the phase transitions
- $\cdots$

$\Longrightarrow$Fast, fully-nonlinear quenches are **universal**!

Consider fully-backreacted quench of $\Delta = 3$ near the boundary:

$$\phi = p_0\rho + \rho^2 p_0' + \rho^3\left(p_{2,0} + \left(\frac{1}{6}p_0^3 + \frac{1}{2}p_0''\right)\ln\rho\right)$$

$$+\rho^4\left(p_{2,0}' - \frac{1}{3}p_0''' + \left(\frac{1}{2}p_0^2 p_0' + \frac{1}{2}p_0'''\right)\ln\rho\right) + \mathcal{O}(\rho^5\ln\rho)$$

$$\Sigma = \frac{1}{\rho}\left(1 - \frac{1}{12}\rho^2 p_0^2 + \mathcal{O}(\rho^3)\right)$$

$$A = \frac{1}{\rho^2}\left(1 - \frac{1}{6}\rho^2 p_0^2 + \mathcal{O}(\rho^4\ln\rho)\right)$$

where

$$p_0 = p_0\left(\frac{v}{\alpha}\right)$$

$\Longrightarrow$ Consider a scaling symmetry $\alpha \to 0$, i.e., introduce

$$t \equiv \frac{v}{\alpha}$$

Then:

$$' \equiv \frac{1}{\alpha}\,\partial_t$$

If above scaling is accompanied by

$$\rho \to r\alpha$$

Above asymptotic expansions have a homogeneous scaling in the limit $\alpha \to 0$

$$\phi \to \alpha\,\phi\,, \qquad \Sigma \to \alpha^{-1}\Sigma\,, \qquad A \to \alpha^{-2}A$$

provided

$$\hat{p}_{2,0} \equiv p_{2,0} + \frac{1}{2}\ln\alpha\; p_0'' \qquad \to \qquad \alpha^{-2}\hat{p}_{2,0}$$

$\Longrightarrow$ Such rescaling can be understood at the level of non-linear equations, not just the asymptotic solution; it:

- effectively linearizes equations of motion
- neglects the scalar backreation on the geometry
- it has a simple physical interpretation...

$\Longrightarrow$ Note: the scaling explains the universal curve for the fast quench of $\Delta = 3$ operator! Previously:

$$\hat{p}_{1,2} \equiv \alpha^2 \ \left(p_{1,2} - \mathcal{F}(\alpha)\ p''_{1,0}\right) ,$$

$$\mathcal{F}\Big|_{fit} \simeq 0.43 - 0.50 \ \ln \alpha$$

while the scaling predict that

$$\hat{p}_{1,2} \equiv \alpha^2 \ \left(p_{1,2} + \frac{1}{2} \ln \alpha \ \ p''_{1,0}\right) ,$$

is a universal curve.

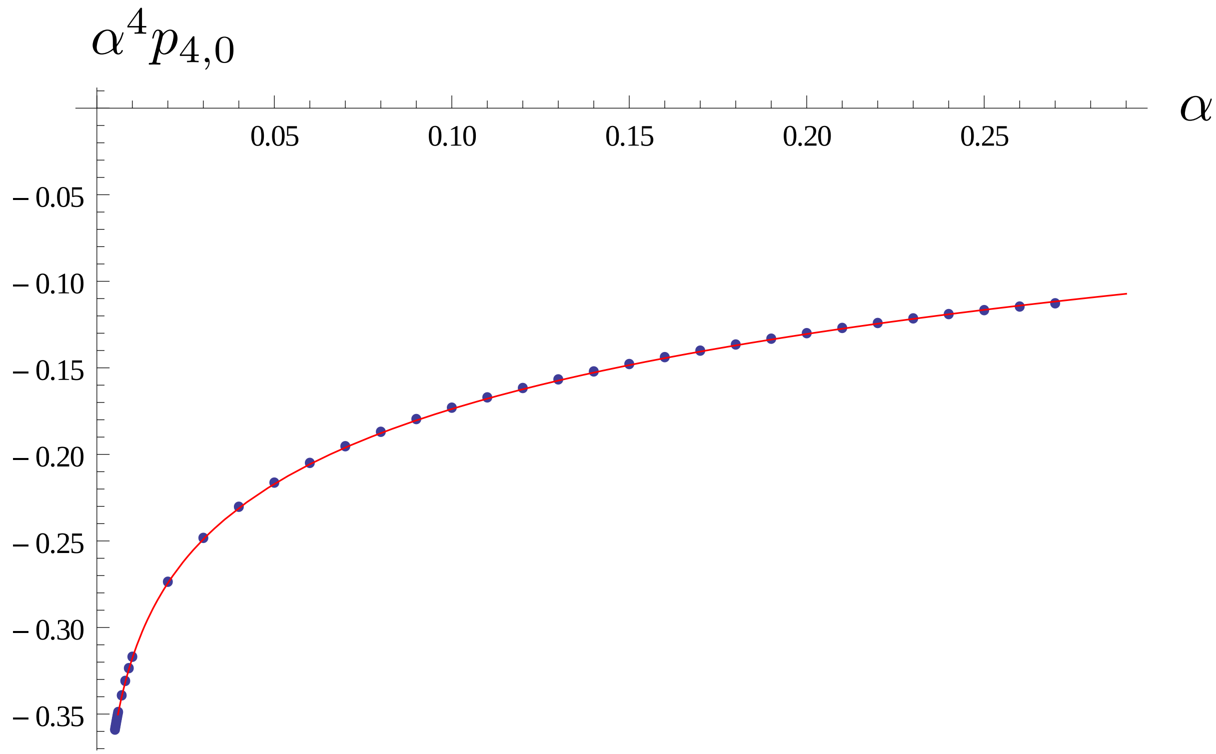
$\Longrightarrow$ Similar arguments can be made for any Δ. For example, for a marginal operator, we have a prediction

$$\hat{p}_{4,0} \equiv p_{4,0} - \frac{1}{16} \ln \alpha \, \frac{1}{\alpha^4} \, \partial_t^4 p_0 \qquad \to \qquad \alpha^{-4} \, \hat{p}_{4,0}$$

or in other words:

$$\alpha^4 p_{4,0} = \frac{1}{16} \ln \alpha \, \partial_t^4 p_0 + \text{const}$$

Results of the quench:



The red line is

$$-0.029 + \frac{1}{16}\ \ln\alpha$$

$\Longrightarrow$ A Fit of the first 20 blue points gives $\ln\alpha$ coefficient:

$$0.0624986\,, \qquad \text{note}: \qquad \frac{1}{16} = 0.0625$$

$\Longrightarrow$ Time-ordered correlator of SHO for abrupt quench

$$C_{\beta_0}(t_1, t_2) = \langle \mathcal{T}\{\phi(t_1)\phi(t_2)\}\rangle|_{\beta_0}$$

From EOM

$$\ddot{\phi} + \omega^2 \phi = 0$$

we have

$$\phi(t) = \phi(0)\ \cos\omega t + \pi(0)\frac{\sin\omega t}{\omega}$$

For $t_1 > t_2$,

$$\langle\phi(t_1)\phi(t_2)\rangle|_{\beta_0} = \langle\phi(0)^2\rangle_{\beta_0}\cos\omega t_1\cos\omega t_2 + \langle\pi(0)^2\rangle_{\beta_0}\frac{\sin\omega t_1\sin\omega t_2}{\omega^2}$$

$$+\langle\phi(0)\pi(0) + \pi(0)\phi(0)\rangle_{\beta_0}\ \frac{\sin\omega(t_1+t_2)}{2\omega} - i\ \frac{\sin\omega(t_1-t_2)}{2\omega}$$

Using

$$\langle \phi(0)^2 \rangle_{\beta_0} = \frac{1}{2\omega_0} \coth \frac{\beta_0 \omega_0}{2}, \qquad \langle \pi(0)^2 \rangle_{\beta_0} = \frac{\omega_0}{2} \coth \frac{\beta_0 \omega_0}{2}$$

$$\langle \phi(0)\pi(0) + \pi(0)\phi(0) \rangle_{\beta_0} = 0$$

We find

$$C_{\beta_0}(t_1, t_2) = \frac{1}{2\omega} e^{-i\omega|t_1 - t_2|} + \left[\frac{\omega_0}{4} \left(\frac{1}{\omega_0^2} + \frac{1}{\omega^2} \right) \coth \frac{\beta_0 \omega_0}{2} - \frac{1}{2\omega} \right] \cos \omega (t_1 - t_2)$$

$$+ \frac{\omega_0}{4} \left(\frac{1}{\omega_0^2} - \frac{1}{\omega^2} \right) \coth \frac{\beta_0 \omega_0}{2} \cos \omega (t_1 + t_2)$$